NEUTRINO EMISSIVITY IN THE QUARK-HADRON MIXED PHASE OF NEUTRON STARS

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Compact Stars in the QCD Phase Diagram VII

11 — 15 June 2018

COLLABORATORS

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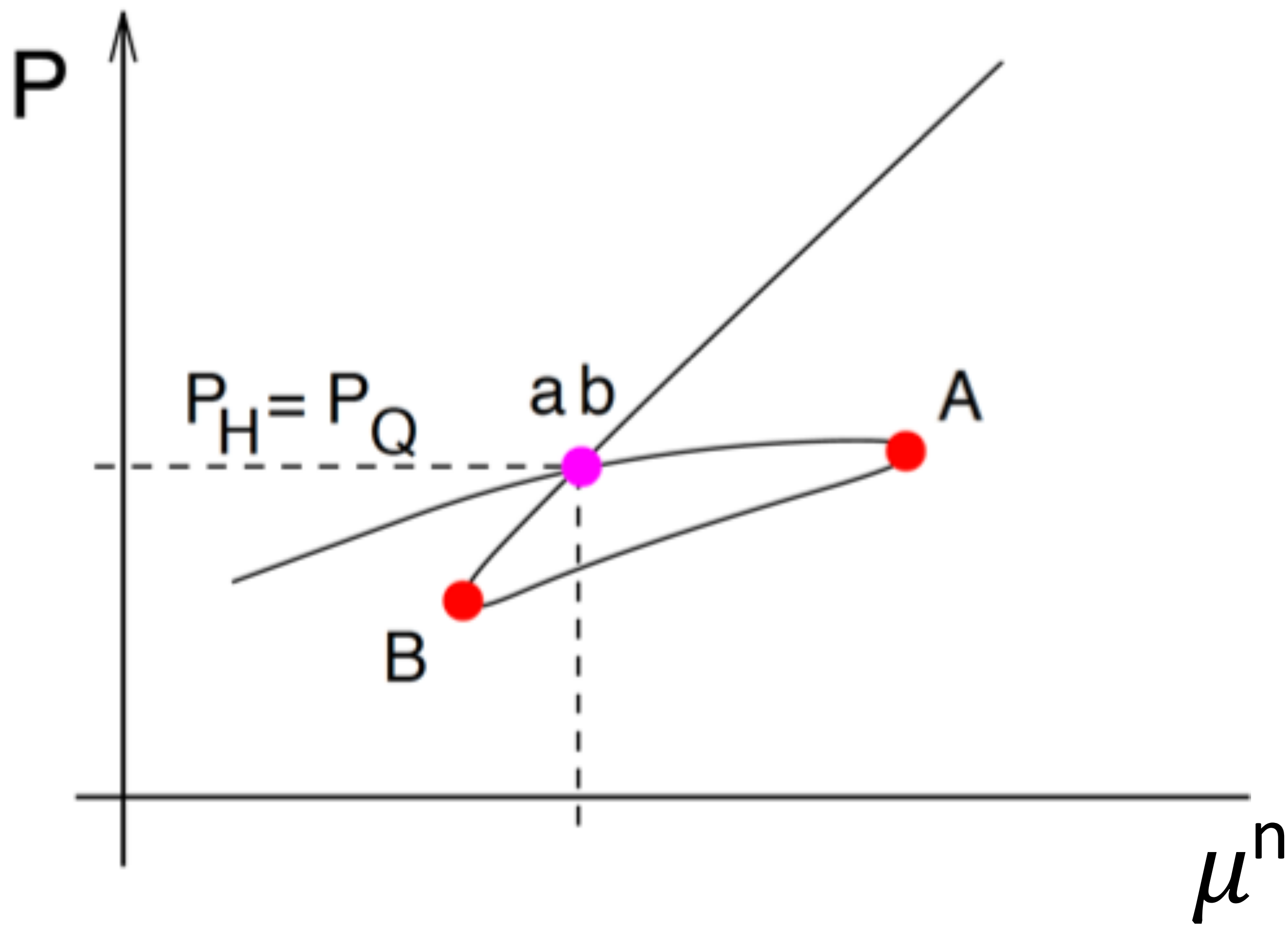
OUTLINE

- Conserved charges
- Quark Deconfinement in Neutron Stars
- Quark-Hadron Lattice
- Electron—Quark Blob Scattering
- Neutrino Emissivity
- Summary

Phase Transition — one conserved charged

Maxwell transition

$$P_{\mathrm{H}}(\mu_{\mathrm{n}}, \{\psi\}) = P_{\mathrm{Q}}(\mu_{\mathrm{n}}, \{q\})$$



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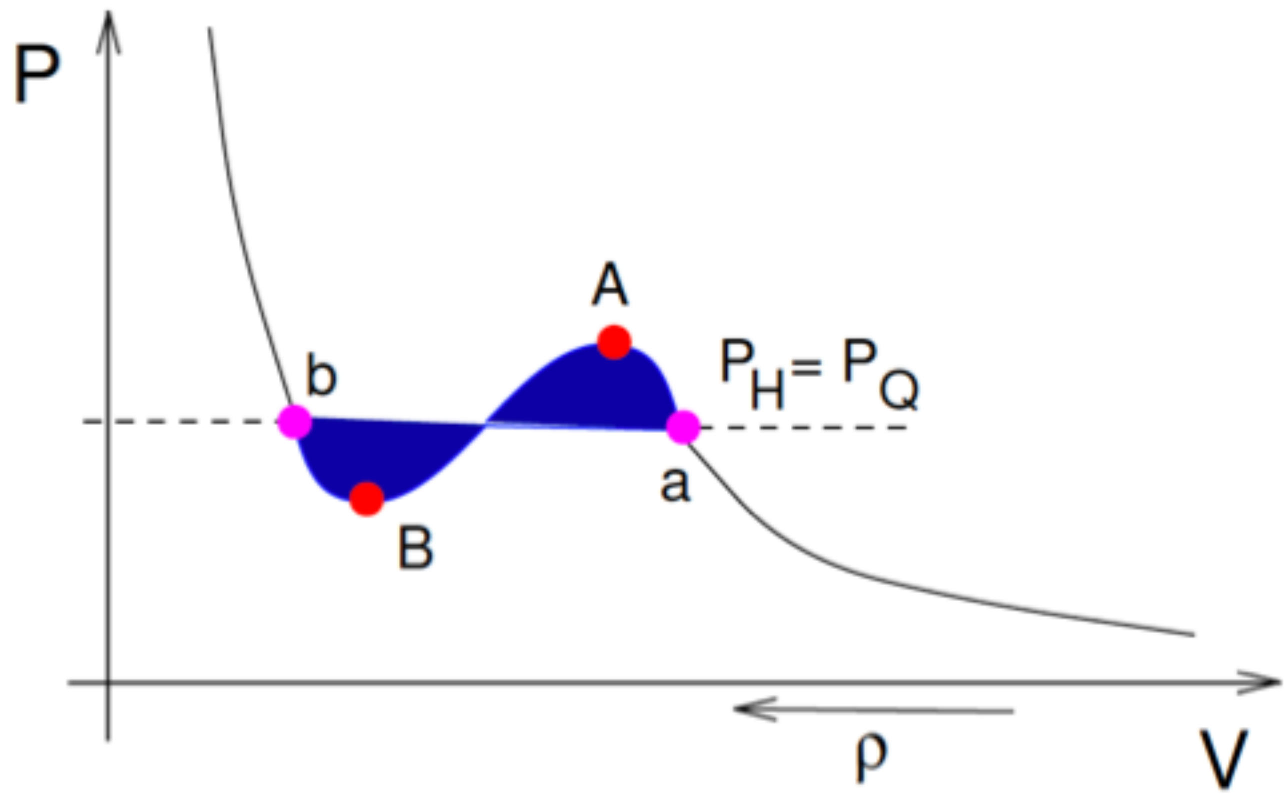
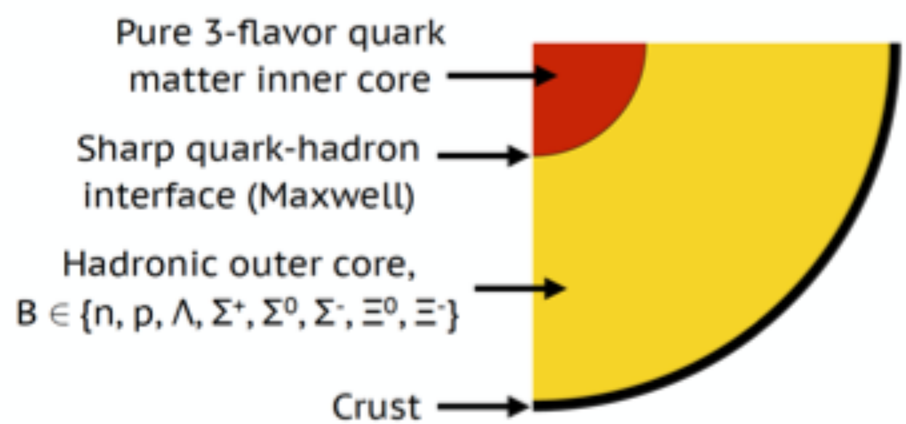


Quark Deconfinement — **Maxwell** Transition

- If surface tension between hadronic and quark matter phases is high ($\alpha > 70$ MeV/fm^2)
  - Constant pressure phase transition favored
  - Maxwell condition for phase equilibrium

    $$P_H(\mu_n, T=0) = P_Q(\mu_n, T=0)$$
  - Local charge neutrality
  - Sharp interface between pure hadronic matter and pure quark matter

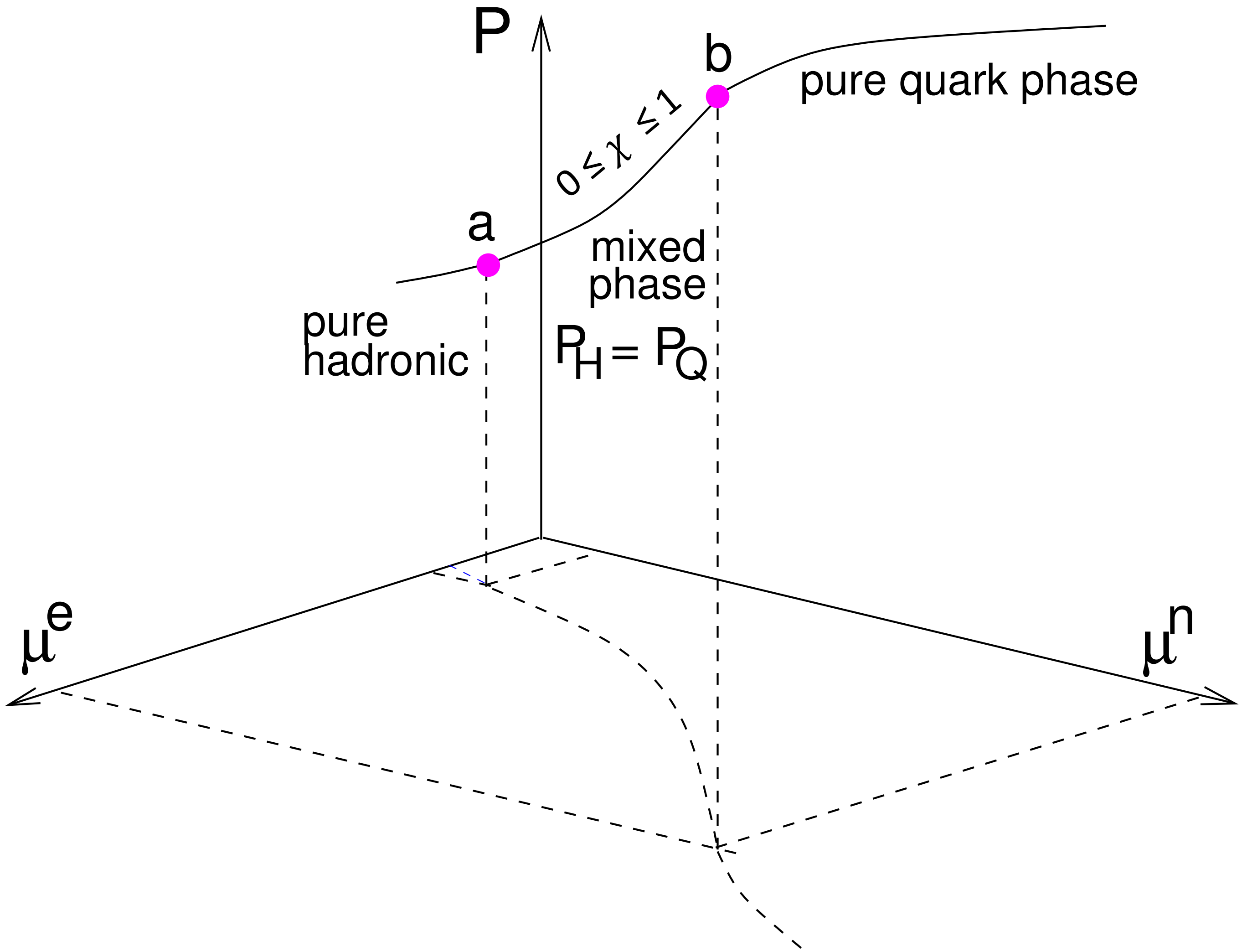


Phase Transition — two conserved charges

Gibbs transition

Glendenning, PRD 46 (1992) 1274
Phys. Rept. 342 (2001) 393

$$P_{\mathrm{H}}(\mu_{\mathrm{n}}, \mu_{\mathrm{e}}, \{\psi\}) = P_{\mathrm{Q}}(\mu_{\mathrm{n}}, \mu_{\mathrm{e}}, \{q\})$$



Quark Deconfinement— **Gibbs** Transition

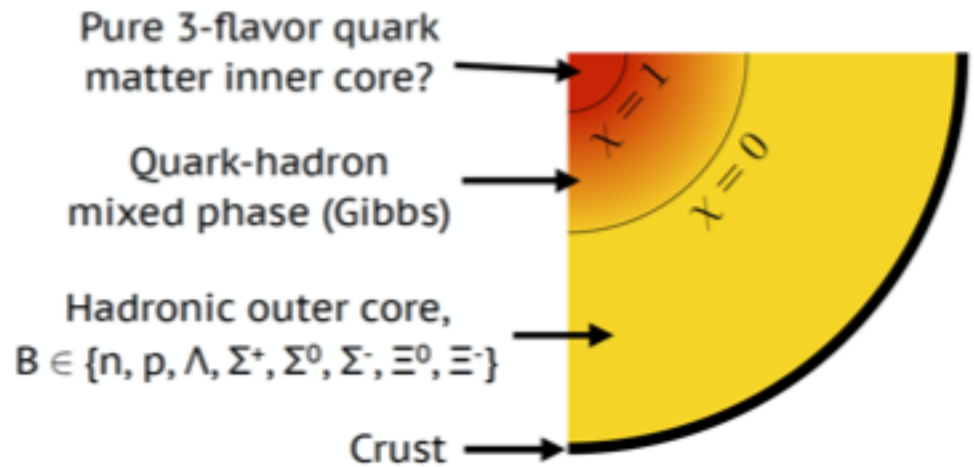
- If surface tension between hadronic and quark matter phases is low ($\alpha < 70$ MeV/fm^2)
  - Gibbs condition for phase equilibrium

    $$P_H(\mu_n,\mu_e,T=0) = P_Q(\mu_n,\mu_e,T=0)$$

  - Global charge neutrality
  - Quark-hadron mixed phase

    $$\chi = V_Q/V \longrightarrow 0 < \chi < 1$$

  - Must simultaneously solve hadronic and deconfined quark nonlinear systems



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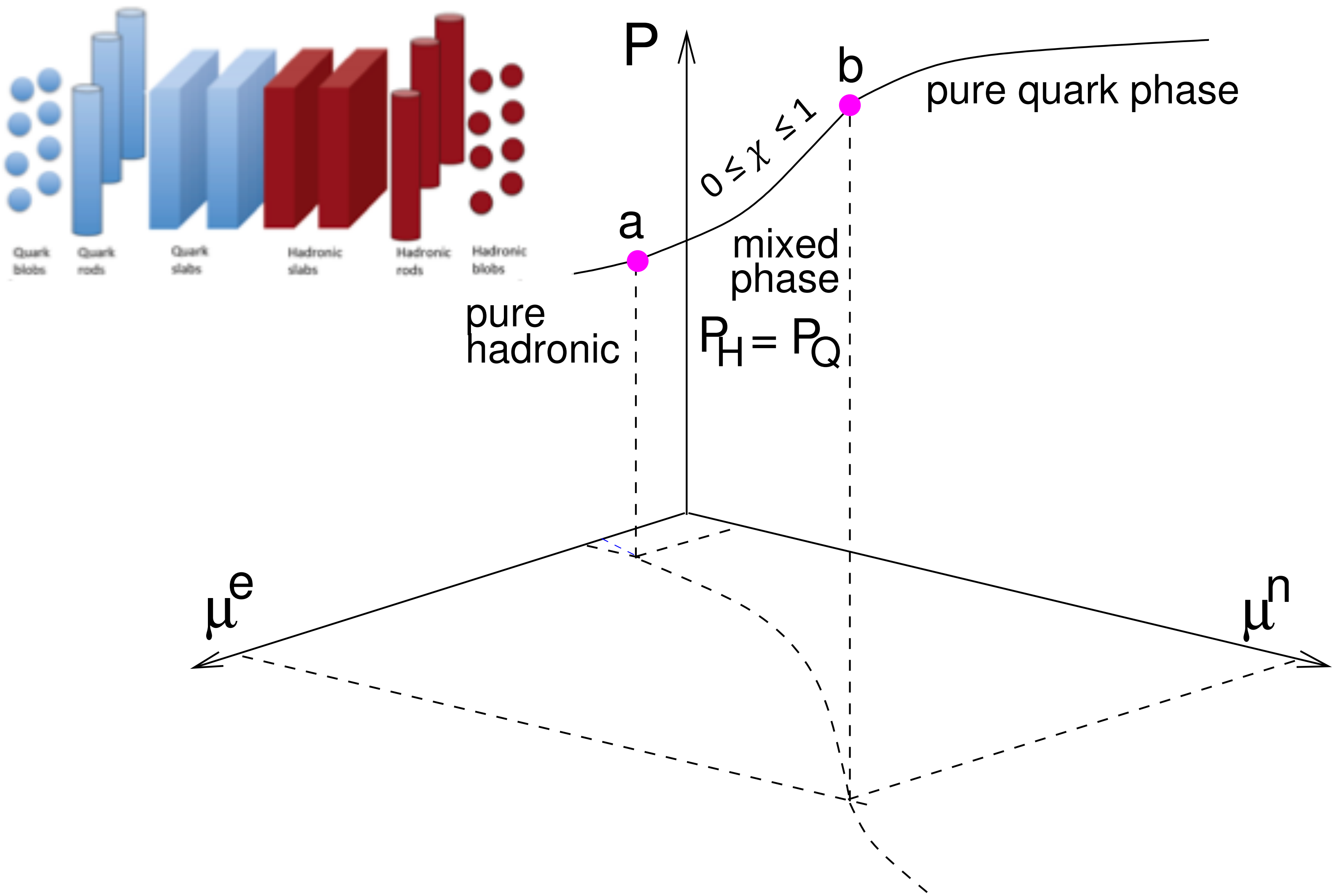

Phase Transition — two conserved charges

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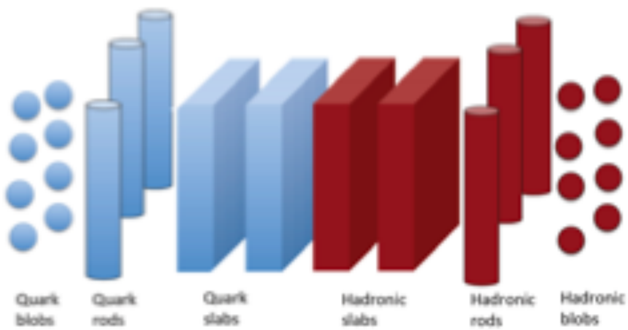
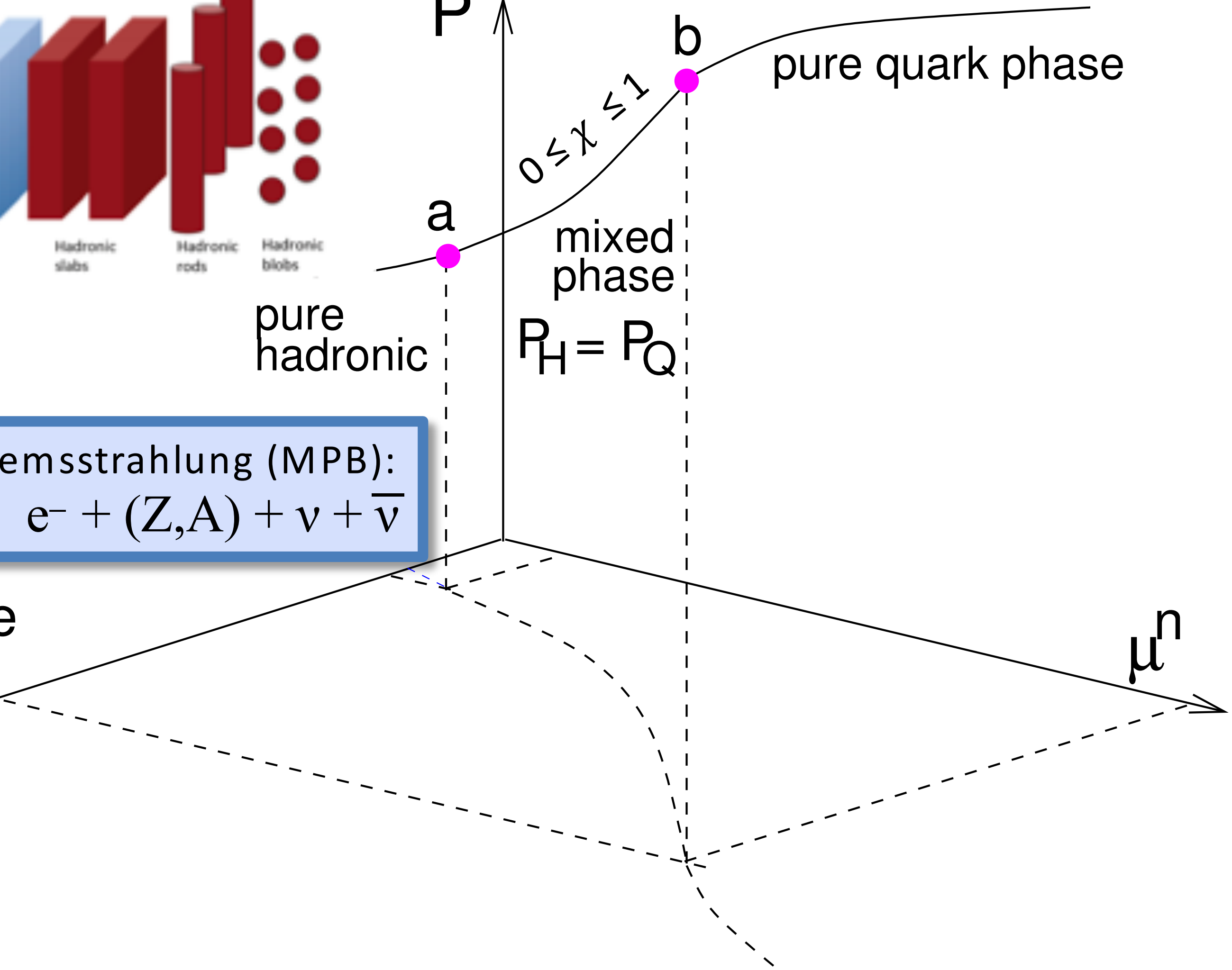
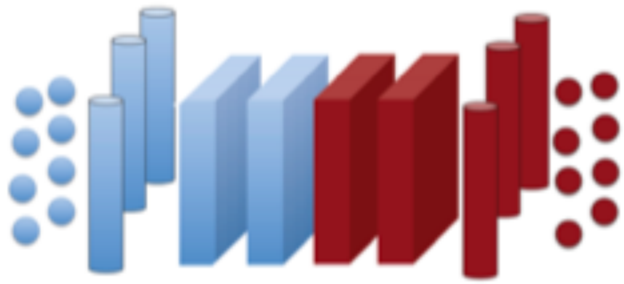




Mixed Phase Bremsstrahlung (MPB):
$e^- + (Z,A) \longrightarrow e^- + (Z,A) + \nu + \bar{\nu}$

Nuclear Lattice Case (electrons + heavy atomic nuclei)

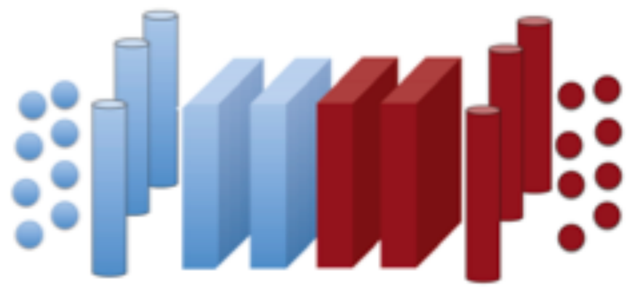


Lamb, Lattimer, Pethick, Ravenhall (1981)
Ravenhall, Pethick, Wilson (1983)
Williams, Koonin (1985)

- Density range: 10^{14} g/cm^3 $\lesssim \rho \lesssim 1.5 \times 10^{14}$ g/cm^3
- Shapes: spheres, rods, slabs
- Electron chemical potential: $\mu_e \sim 80$ MeV
- Atomic number: $Z \sim 50$
- Radius of Wigner-Seitz cell: $R \sim 18$ fm
- Radius of rare phase structure: $r \sim 9$ fm
- Melting temperature: $T_{melt} = (Z e)^2 / (R k_B \Gamma_{melt}) \sim 1.3 \times 10^{10}$ K

Haensel Kaminker, Yakovlev (1996)
Yakovlev, Kaminker (1996)
Kaminker, Pethick, Potekhin, Thorsson, Yakovlev (1999)

Quark-Hadron Lattice Case (blobs)



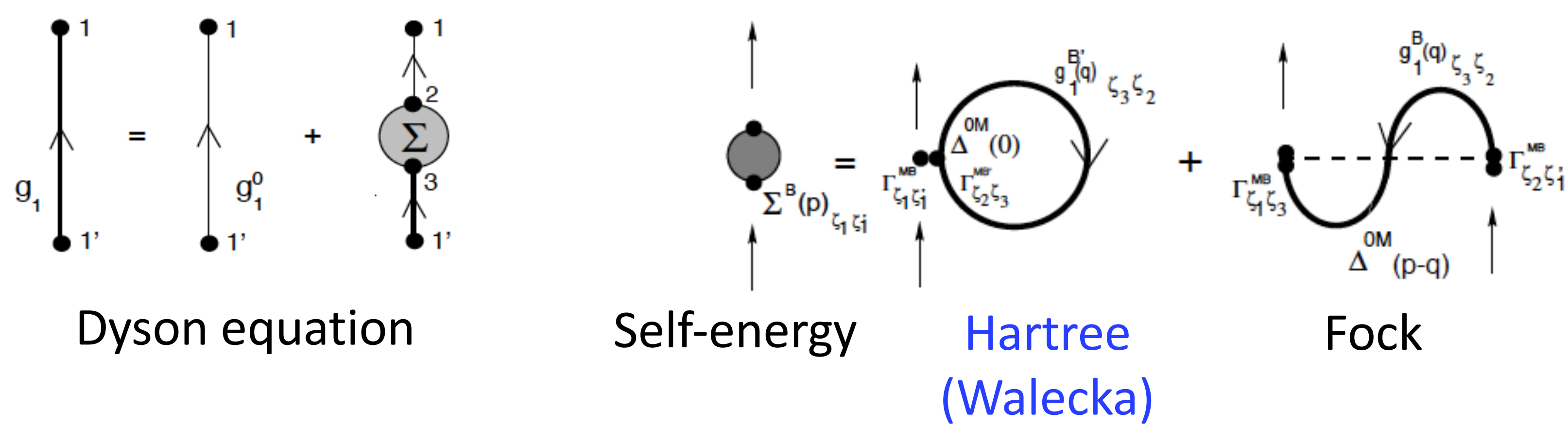
- Density range: $\rho \gtrsim 7.5 \times 10^{14}$ g/cm^3 (3 times nuclear saturation)
- Electron chemical potential: $\mu_e \sim 140$ MeV
- Atomic number: $Z \sim 200$
- Radius of Wigner-Seitz cell: $R \sim 12$ fm
- Radius of quark blob: $r \sim 8$ fm
- Melting temperature: $T_{melt} = (Z\,e)^2 / (R\,k_B\,172) \sim 4 \times 10^{11}$ K

Models for the Nuclear EOS

$$\mathcal{L}_{\rm NM} = \sum_B \bar{\psi}_B \Big[\gamma^\mu \Big(i\partial_\mu - g_{\omega B}\omega_\mu - \frac{1}{2} g_{\rho B} \vec{\tau}\cdot\vec{\rho}_\mu\Big) - (m_B - g_{\sigma B}\sigma)\Big]\psi_B$$

$$+\frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu$$

$$-\frac{1}{4}\vec{\rho}^{\mu\nu}\vec{\rho}_{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}^\mu\cdot\vec{\rho}_\mu + \sum_\lambda \bar{\psi}_\lambda \left(i\gamma^\mu\partial_\mu - m_\lambda\right)\psi_\lambda$$



Hadronic Equation of State

- **Standard RMF** nonlinear system

meson field equations

$$m_\sigma^2\bar{\sigma} + b_\sigma m_N g_{\sigma N}\left(g_{\sigma N}\bar{\sigma}\right)^2 + c_\sigma g_{\sigma N}\left(g_{\sigma N}\bar{\sigma}\right)^3 - \sum_B g_{\sigma B} n_B^S = 0$$

$$m_\omega^2\bar{\omega} - \sum_B g_{\omega B} n_B = 0$$

$$m_\rho^2\bar{\rho} - \sum_B g_{\rho B} I_{3B} n_B = 0$$

$$\sum_B n_B q_B - \sum_\lambda n_\lambda = 0$$

$$n - \sum_B n_B = 0$$

$$\mu_B - (\mu_n - q_B\mu_e) = 0$$

- System variables: $\bar{\sigma},\ \bar{\omega},\ \bar{\rho},\ k_n,\ k_e$
- Meson-baryon coupling constants: $g_{iB}(n) = g_{iB}(n_0),\ b_\sigma,\ c_\sigma$

- **DDRMF**: density-dependent meson-baryon coupling constants

$$m_\sigma^2\bar{\sigma} - \sum_B g_{\sigma B} n_B^S = 0$$
$$m_\omega^2\bar{\omega} - \sum_B g_{\omega B} n_B = 0$$
$$m_\rho^2\bar{\rho} - \sum_B g_{\rho B} I_{3B} n_B = 0$$
$$\sum_B n_B q_B - \sum_\lambda n_\lambda = 0$$
$$n - \sum_B n_B = 0$$
$$\mu_B - (\mu_n - q_B\mu_e) = 0$$

- System variables: $\bar{\sigma},\ \bar{\omega},\ \bar{\rho},\ k_n,\ k_e,\ k_B,\ \ B \in \{p,\ \Lambda,\ \Sigma,\ \Xi,\ \Delta\}$
- Meson-baryon coupling constants: $g_{iB}(n) = g_{iB}(n_0)h_i(x),\ x = n/n_0$

$$h_i(x) = a_i \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2},\ \ i \in \{\sigma,\ \omega\} \qquad h_\rho(x) = \exp\left[-a_\rho\left(x - 1\right)\right]$$

Deconfined Quark Matter

- Deconfined quark matter modeled using the nonlocal three-flavor Nambu-Jona-Lasinio (n3NJL) model
- Thermodynamic potential Ω^{NL}

$$\Omega^{\rm NL}(M, T=0, \mu) = -\frac{3}{\pi^3}\sum_f \int_0^\infty dk_0 \int_0^\infty dk\, \ln\left\{\left[\hat{p}_f^2 + M_f^2(p_f)\right]\frac{1}{p_f^2 + m_f^2}\right\}$$
$$-\frac{3}{\pi^2}\sum_f \int_0^{\sqrt{\mu_f^2 - m_f^2}} dk\, k^2 \left[(\mu_f - E_f)\,\theta(\mu_f - m_f)\right]$$
$$-\frac{1}{2}\left[\sum_f \left(\bar{\sigma}_f \bar{S}_f + \frac{G_S}{2}\bar{S}_f^2\right) + \frac{H}{2}\bar{S}_u\,\bar{S}_d\,\bar{S}_s\right] - \sum_f \frac{\bar{\omega}_f^2}{4G_V}$$

- n3NJL nonlinear system, $f \in \{u,\, d,\, s\}$

System Variables: $\bar{\sigma}_f,\, \bar{\omega}_f,\, k_n,\, k_e$

$$\frac{\partial\Omega^{\rm NL}}{\partial\bar{\sigma}_f} = \bar{\sigma}_f + G_S\bar{S}_f + \tfrac{1}{2}H\bar{S}_i\bar{S}_j = 0$$

if vector interaction included, $G_V > 0$ ⟶

$$\frac{\partial\Omega^{\rm NL}}{\partial\bar{\omega}_f} = \bar{\omega}_f - 2G_V\frac{\partial\Omega^{\rm NL}}{\partial p_f} = 0$$

$$\sum_f n_f q_f + \sum_\lambda n_\lambda q_\lambda = 0$$

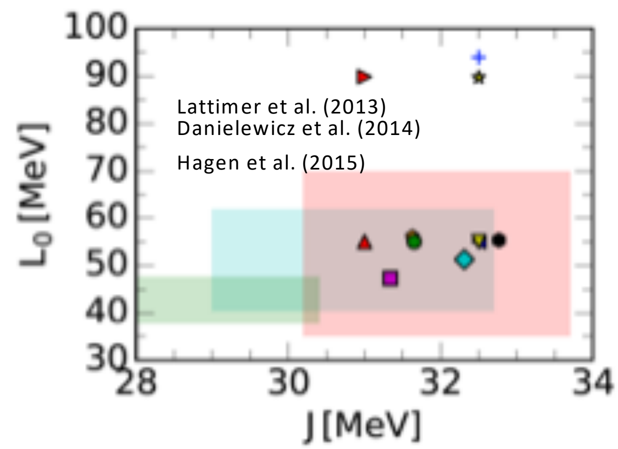
$$n - \tfrac{1}{3}\sum_f n_f = 0$$

M. Orsaria et al; Blaschke et al., Sedrakian et al., Lugones et al., Kashiwa, Hell & Weise, …

Coupling Constants and Constraints on EoS

1. Properties of symmetric nuclear matter at saturation density, n_0

   E/N, m*, K, J, L



2. Hyperons (Y)

   Scalar meson-Y coupling constants: $x_{\sigma Y}$
   - U_Λ=–28 MeV, U_Ξ=–18 MeV, U_Σ=+30 MeV

   Vector meson-Y coupling constants: $x_{\omega Y}$
   - SU(3) flavor symmetry
   - SU(6) spin-flavor symmetry (quark counting model)

     Dover & Gall (1984), Schaffner-Bielich et al. (1994), Weissborn et al. (2013), Miyatsu et al. (2013), Oertel et al. (2015)

   Isovector meson-Y coupling constants: $x_{\rho Y}$
   - Lynch et al. (2009), Weissenborn et al. (2012), Miyatsu et al. (2013)

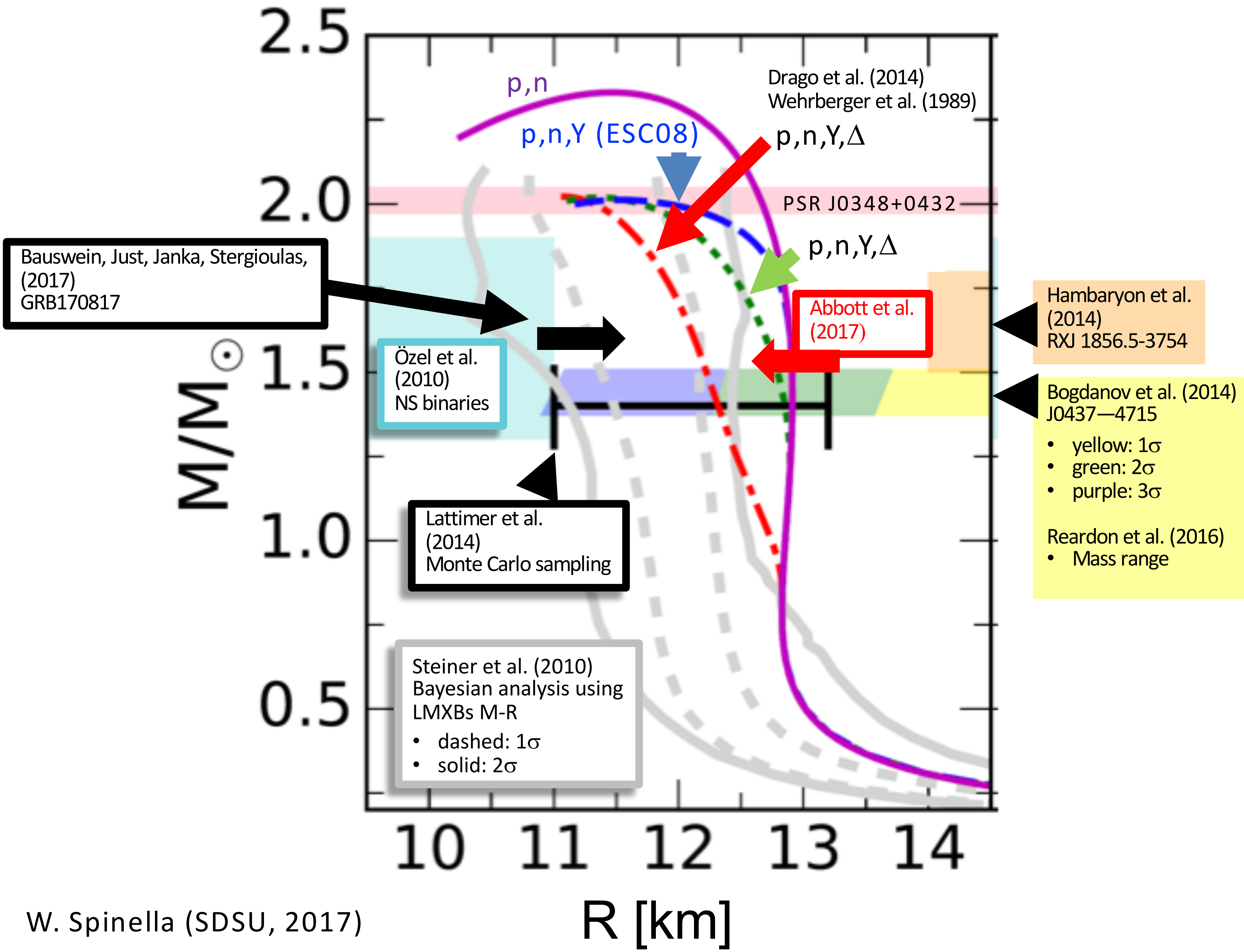
3. Deltas (Δ)

   Coupling constants: $x_{\sigma\Delta}$, $x_{\omega\Delta}$, $x_{\rho\Delta}$
   - Drago et al. (2014): -30 MeV+$U_N(n_0) < U_\Delta(n_0) < U_N(n_0)$,
     $0 < x_{\sigma\Delta} - x_{\omega\Delta} < 0.2$
   - Kolomeitsev et al (2017): $U_N(n_0) < U_\Delta(n_0) < 2U_N(n_0)/3$,
     $U_\Delta(n_0)= -50$ MeV (see also Riek et al. (2009))

     $x_{\sigma\Delta}=x_{\omega\Delta}=1.1$, $x_{\rho\Delta}=1.0$

4. Heavy Ion collisions
   - Lynch et al. (2009), Danielewicz et al. (2002)

Mass-Radius Relationship



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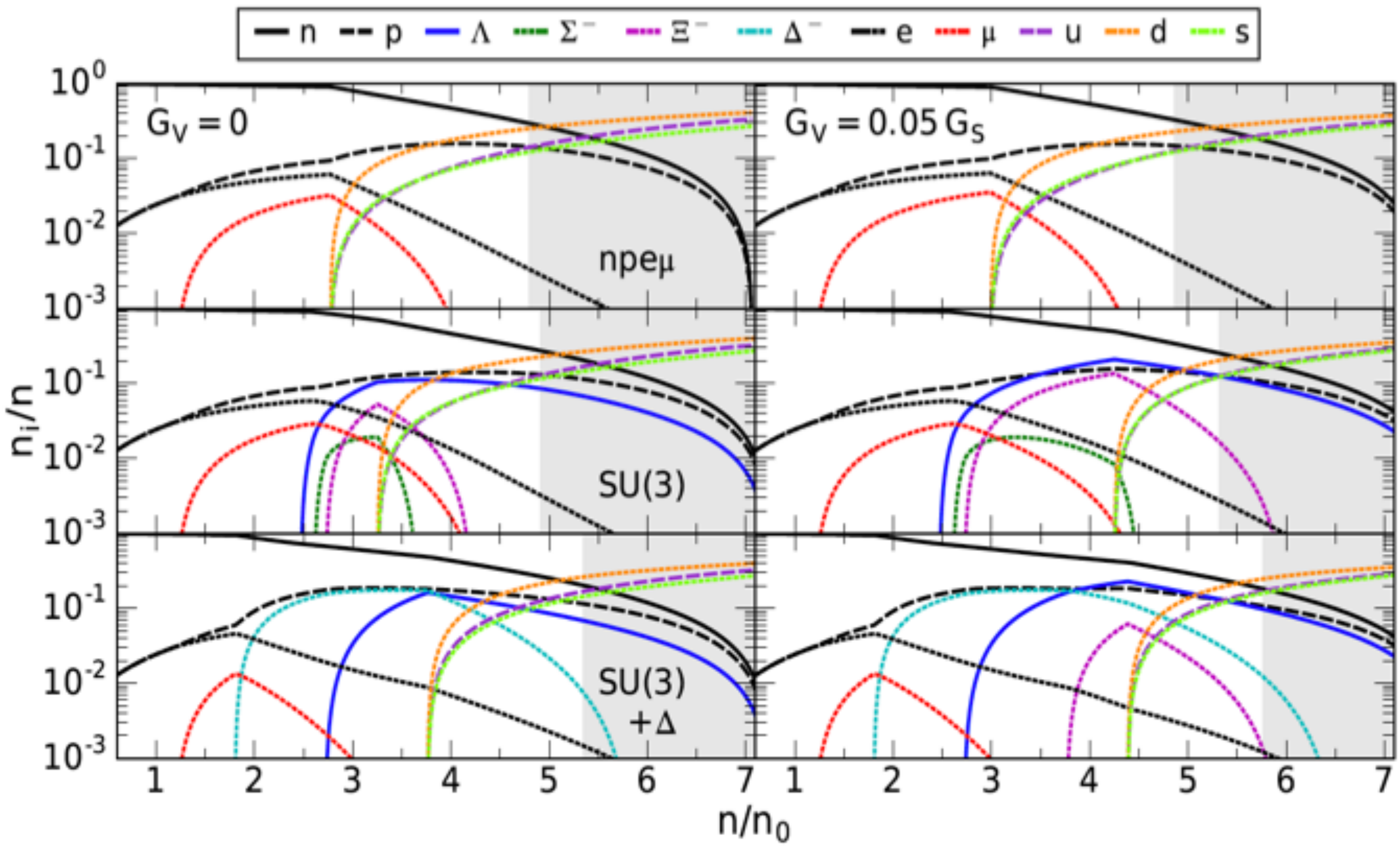
Quark-Hadron Composition

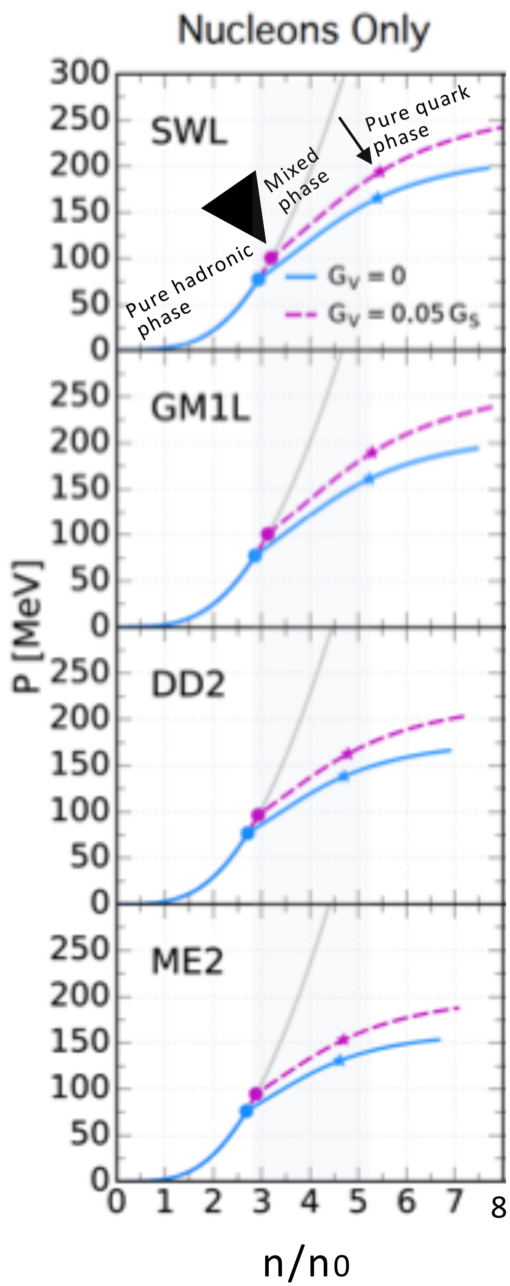
DD2 Parametrization



W. Spinella (SDSU, 2017)

Gibbs Transition

- Onset of quark deconfinement at ~3 to 4.5 n_0



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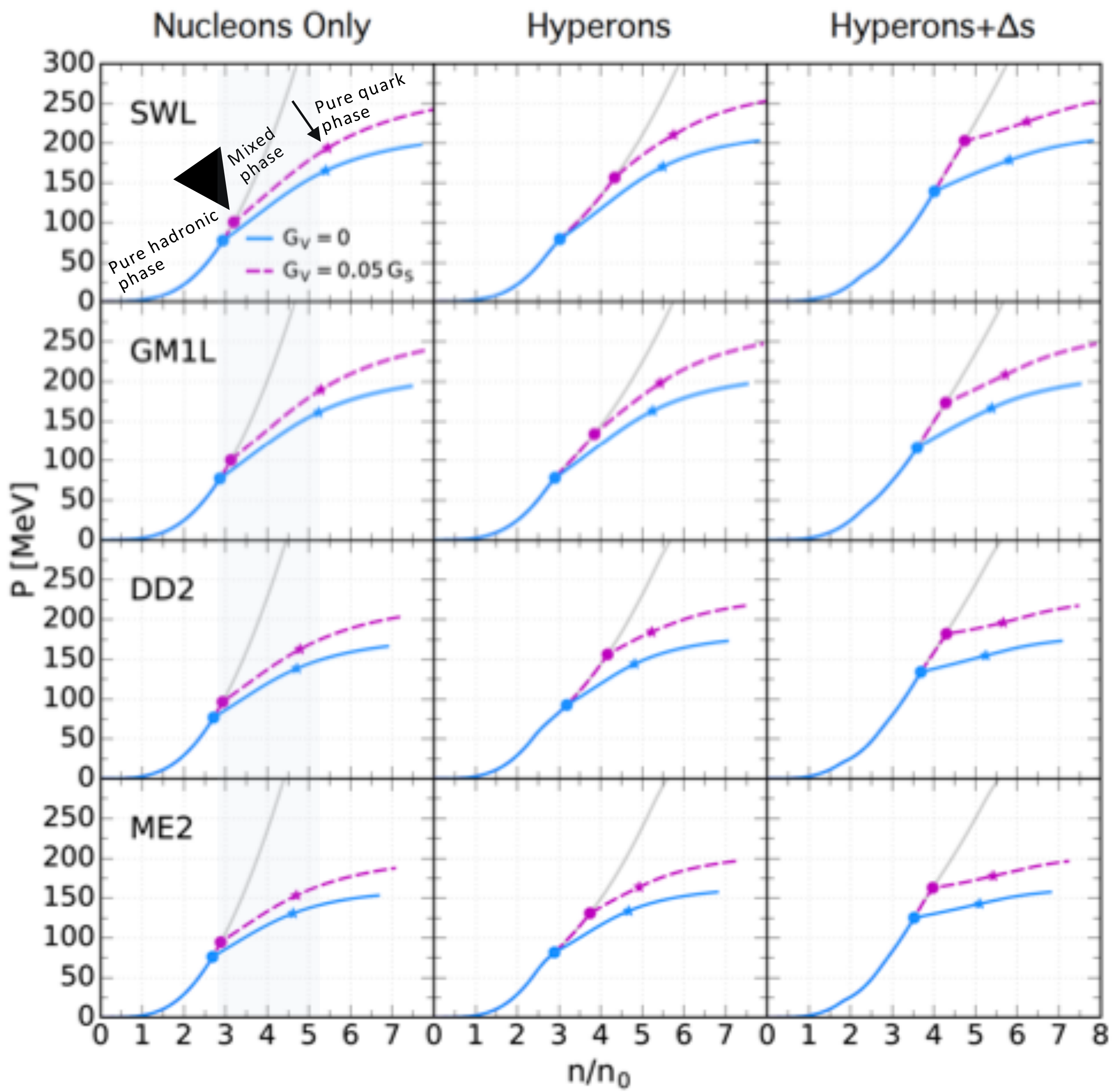
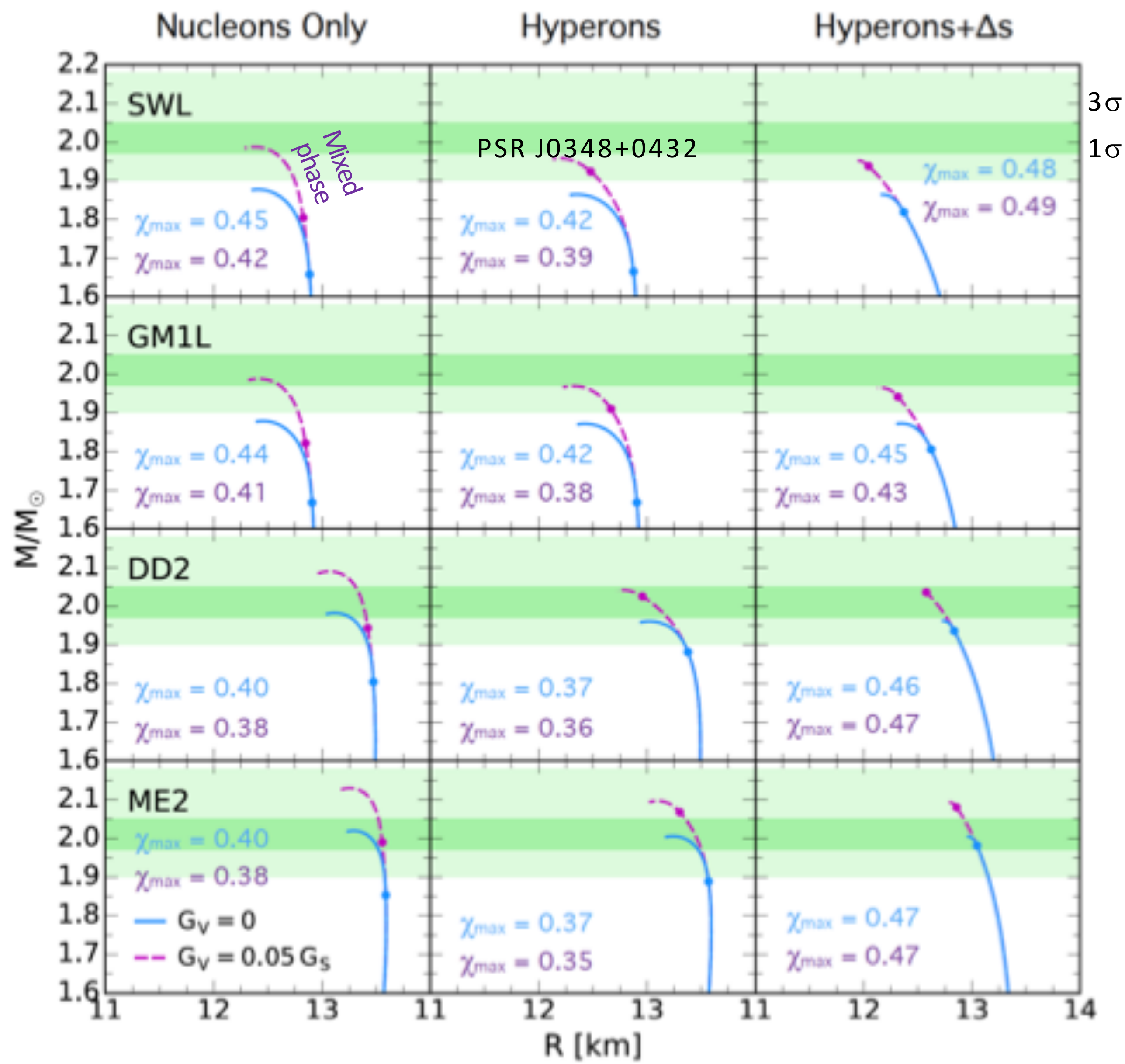


W. Spinella (2017)

Gibbs Transition

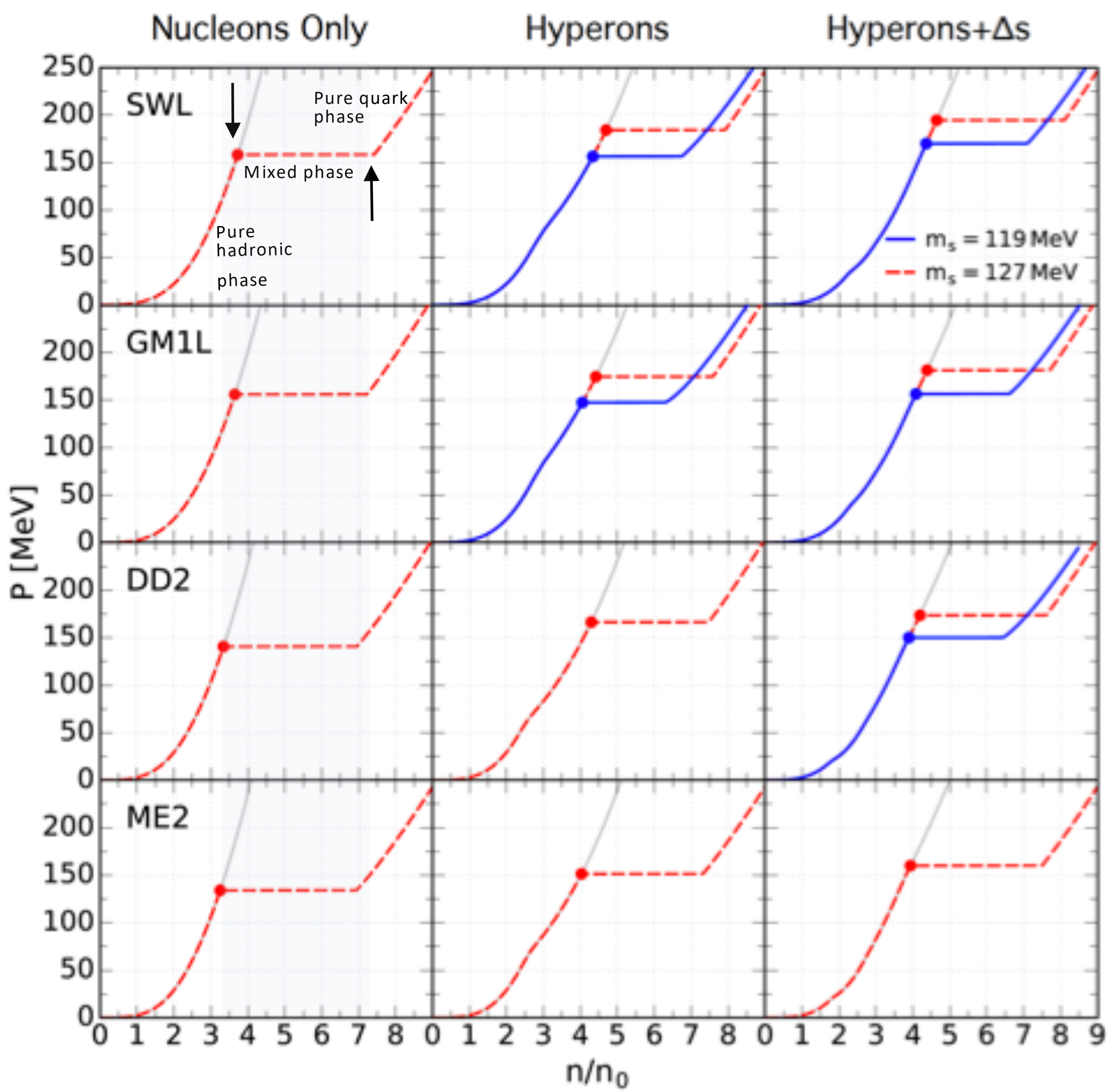


- 35-50% quark matter in mixed phase core

W. Spinella (2017)

Maxwell Transition

- Onset of quark deconfinement at ~3 to 4.5 n_0



W. Spinella, FW (2017)

Back to Quark-Hadron Lattice Roadmap

- Surface tension
- Blob size
- Charge number
- Neutrino emission rate
  - Static lattice contribution
  - Phonon contribution

Surface tension of the quark-hadron interface

$$\alpha(\chi) = \eta L \left[\epsilon_Q(\chi) - \epsilon_H(\chi)\right]$$

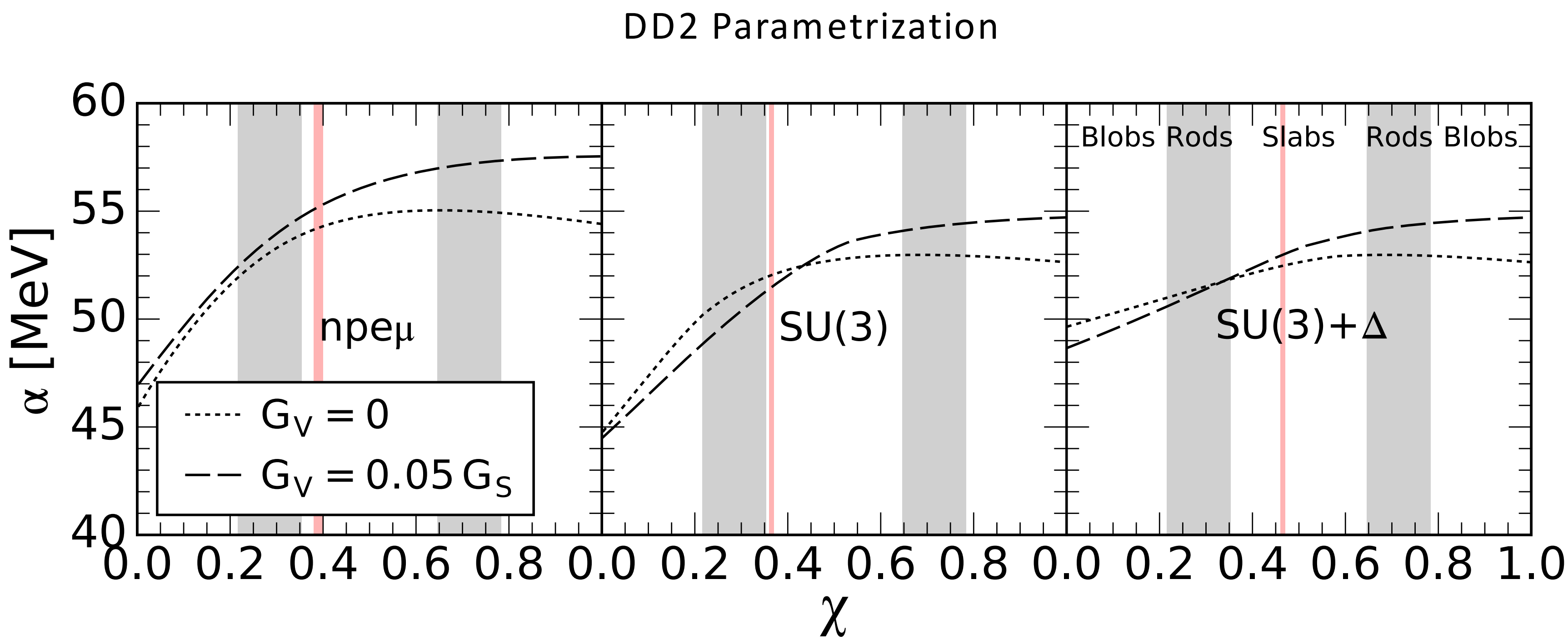
$$L \sim 1 \text{ fm}$$

$$\eta \sim \mathcal{O}(1)$$

$$\alpha \lesssim 55 \text{ MeV fm}^{-2}$$

Yasutake+ (2014), Palhares & Fraga (2010),
Pinto, Koch, Randrup (2012), Mintz+ (2013)

Surface tension α in the quark-hadron mixed phase



Spinella, FW, Orsaria, Contrera (2018)

Coulomb and Surface energy densities

Glendenning '92

$$\epsilon_C = 2\pi e^2 \left[q_H(\chi) - q_Q(\chi)\right]^2 r^2 x f_D(x)$$
$$\epsilon_S = Dx\alpha(\chi)/r$$

$q_H\,(q_Q)$: Hadronic (Quark) phase charge density

r : radius of rare phase structure

D : dimensionality of the lattice (D=1,2,3)

$$f_D(x) = \frac{1}{D+2}\left[\frac{1}{D-2}(2 - Dx^{1-2/D}) + x\right]$$

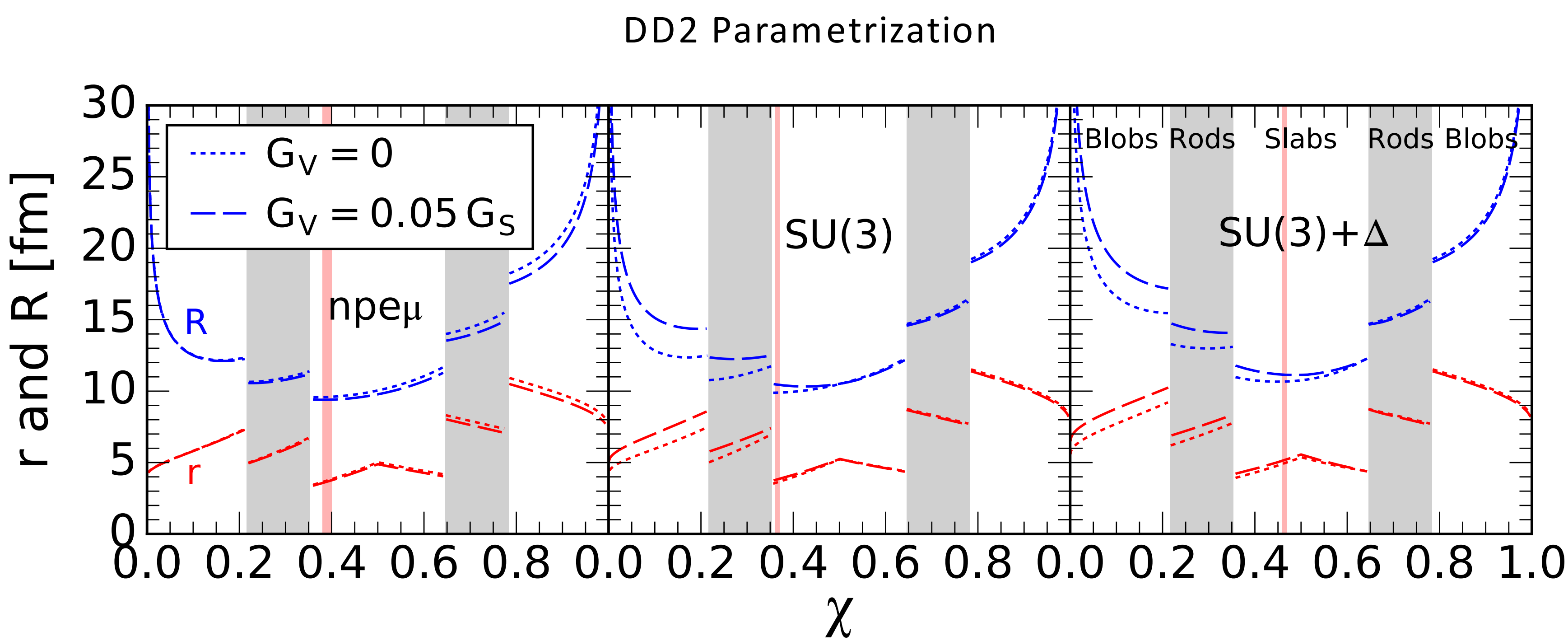
Rare phase structure size, charge, number density

Glendenning '92

Minimizing $\frac{\partial(\epsilon_C + \epsilon_S)}{\partial r}$ leads for the size of the rare phase to:

$$r = \left(\frac{D\alpha(\chi)}{4\pi e^2 f_D(\chi) \left[q_H(\chi) - q_Q(\chi)\right]^2} \right)^{\frac{1}{3}}$$

Radius of the rare phase structure r and Wigner-Seitz cell R in the quark-hadron mixed phase



Spinella, FW, Orsaria, Contrera (2018)

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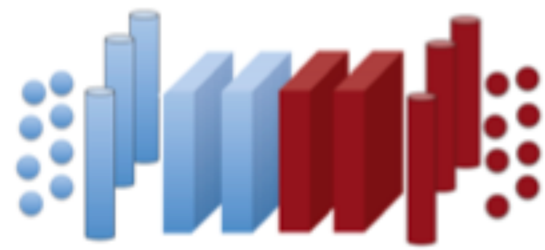


- Surface tension
- Blob size
- Charge number
- Neutrino emission rate
  - Static lattice contribution
  - Phonon contribution

Static Lattice Contribution to Neutrino Emissivity

$$L_{\rm sl} = \frac{1}{12Z} \sum_{\substack{K \neq 0 \\ (K<2k_e)}} \frac{(1-y_K^2)}{y_K^2} \frac{|F(K)|^2}{|\epsilon(K)|^2} I(y_K, t_V)\, e^{-2W(K)}$$

Pethick & Thorsson (1999)
Kaminker+ (1999)

$$J_{\rm sl} = \sum_{K \neq 0} \frac{y_K^2}{t_V^2} I(y_K, t_V)$$

$W(K)$: Debye-Waller factor

$I(y_K, t_V)$: see eq. (39) in Kaminker et al., AA 343 (1999) 1009

$$y_K = K/(2k_e)$$

$$t_V = k_B T / \left[|V(K)|(1-y_K^2) \right]$$

Definitions

Debye-Waller factor:

$$W(q) = \begin{cases} \frac{aq^2}{8k_e^2}\left(1.399\, e^{-9.1t_p} + 12.972\, t_p\right) & \text{spherical blobs} \\ 0 & \text{rods and slabs} \end{cases}$$

$$t_p = T/T_p$$

$$T_p = \frac{\hbar\omega_p}{k_b}$$

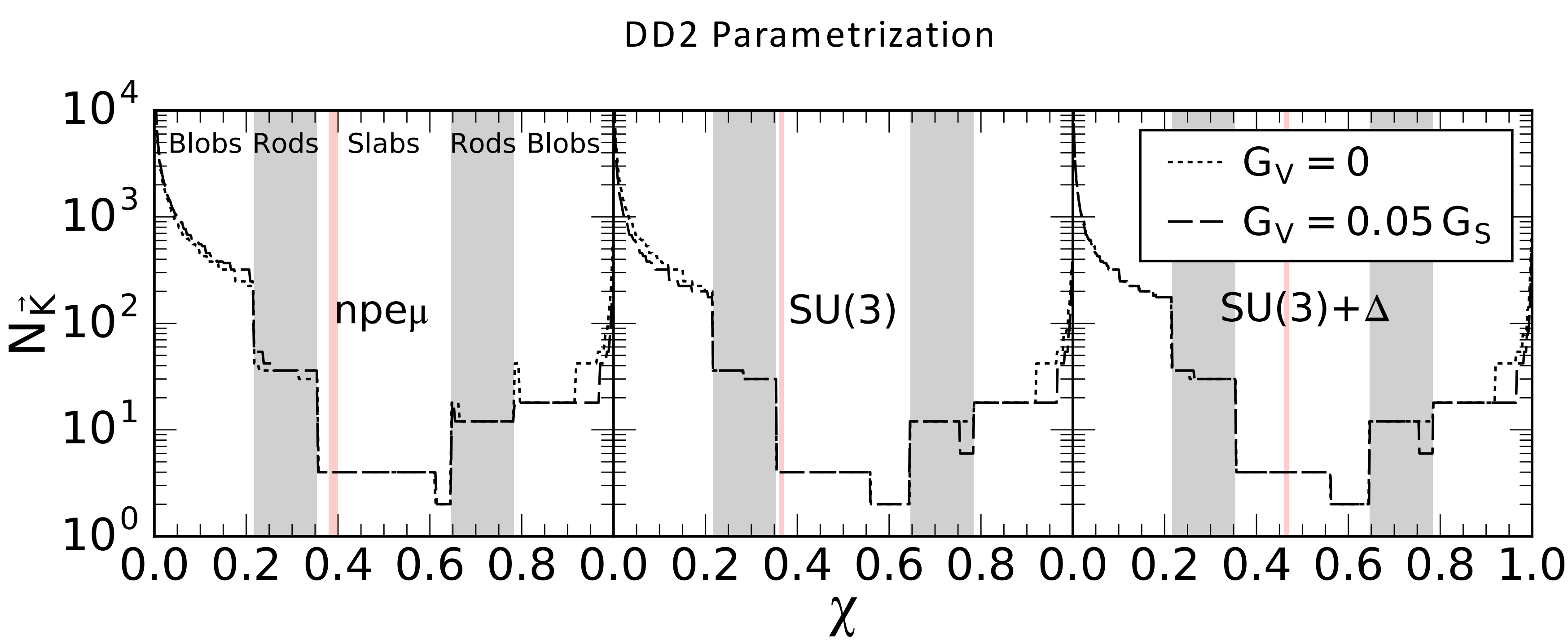
$$\omega_p = \sqrt{\frac{4\pi Z^2 e^2 n_b}{m_b}}$$

$$a = 4\hbar^2 k_e^2/(k_B T_p m_b)$$

$$F(q) = \frac{3}{(qR^3)}\left[\sin(qR) - qR\cos(qR)\right]$$

$$V(q) = \frac{4\pi e \rho_Z F(q)}{q^2 \epsilon(q)} e^{-W(q)}$$

Number of scattering vectors that satisfy the condition $K<2k_e$



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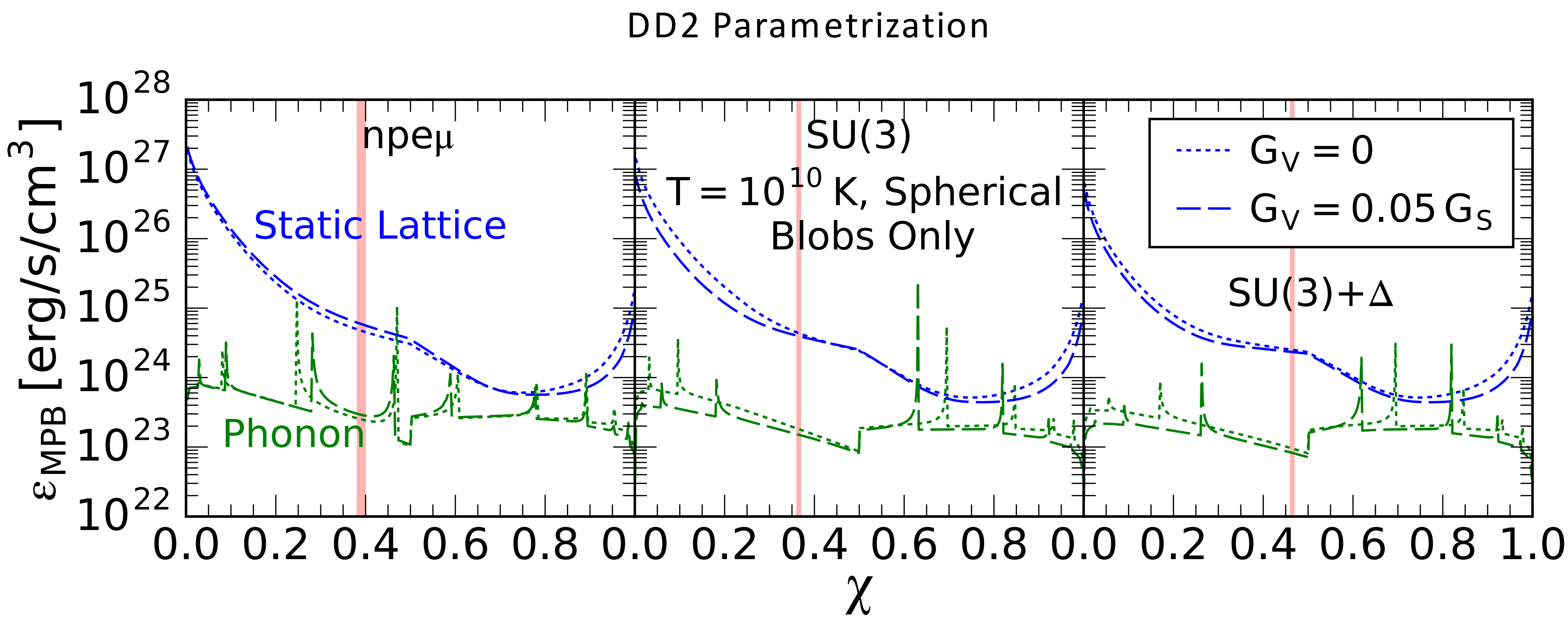
Static lattice and phonon contributions to the neutrino emissivity

T=10^{10} K



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Phonon Contribution to Neutrino Emissivity

$$L_{\rm ph} = \int_{y_0}^{1} dy \frac{S_{\rm eff}(q)|F(q)|^2}{y|\epsilon(q,0)|^2} \left(1 + \frac{2y^2}{1-y^2}\ln y\right)$$

Kaminker+ (1999)

Baiko+ (1998)

Definitions:

$$y = q/(2k_e) \qquad |\boldsymbol{q}| > q_0 \gtrsim (6\pi^2 n_b)^{1/3}$$

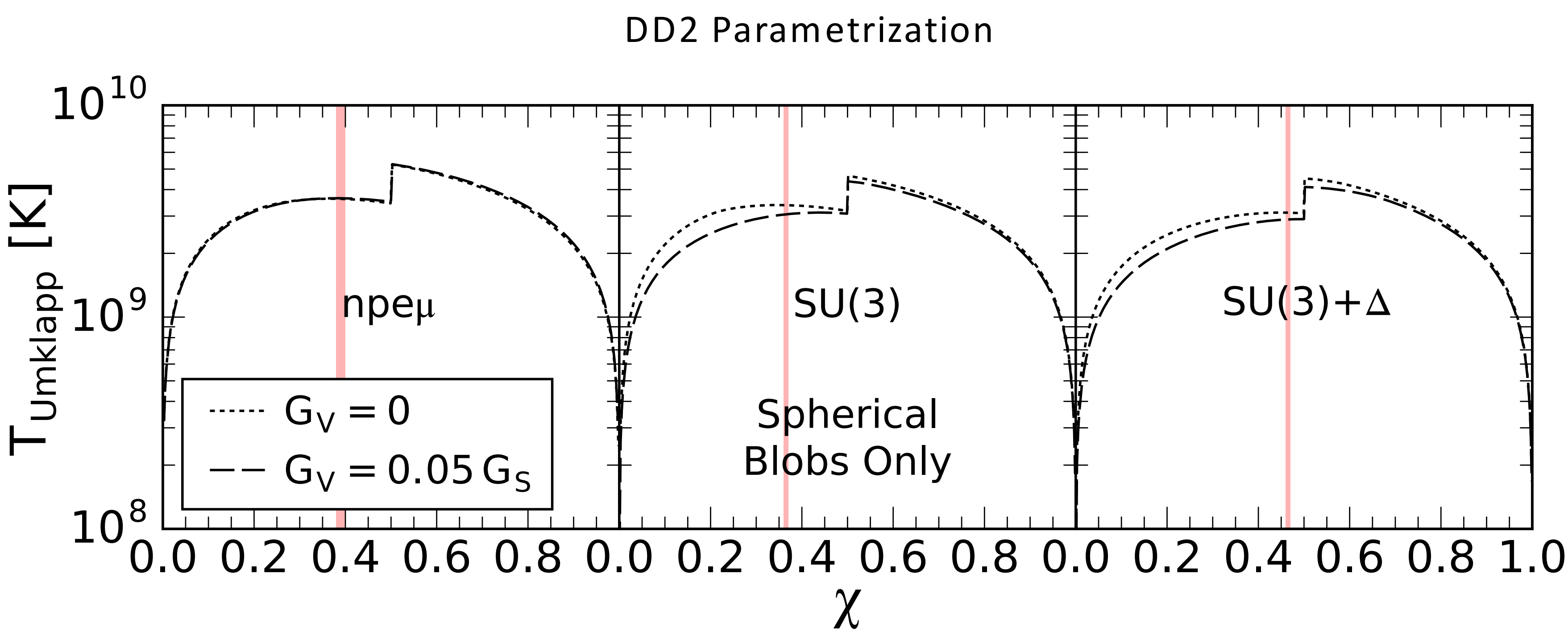
$$S_{\rm eff}(q) = 189\left(\frac{2}{\pi}\right)^5 e^{-2W}\int_0^\infty d\xi\, \frac{1-40\xi^2+80\xi^4}{(1+4\xi^2)^5\cosh^2(\pi\xi)}\left(e^{\Phi(\xi)}-1\right)$$

$$\Phi(\xi) = \frac{\hbar q^2}{2m_b}\left\langle \frac{\cos(\omega_s t)}{\omega_s \sinh(\hbar\omega_s/2k_BT)}\right\rangle$$

$$\xi = tk_BT/\hbar$$

$\langle\ldots\rangle$: averaging over phonon frequencies and phonon modes

Temperature below which Umklapp processes are frozen out



Spinella, FW, Orsaria, Contrera (2018)

Neutrino Emissivity

$$\epsilon_{\rm MPB}^{\rm blobs} \approx 5.37 \times 10^{20}\, n T_9^6 Z^2 L \,\mathrm{erg\,s^{-1}\,cm^{-3}}$$

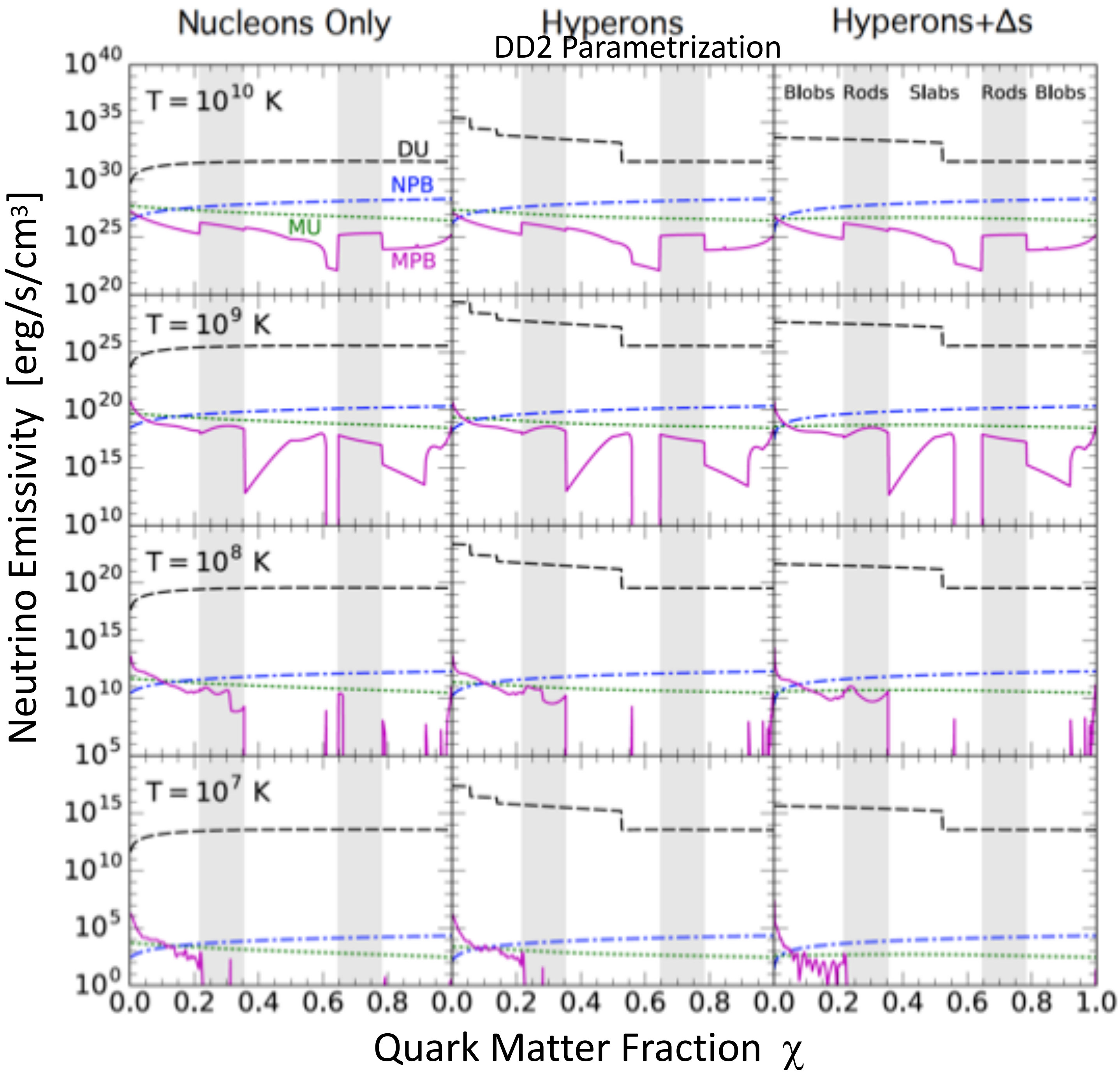
Haensel+ (1999)

$$L = L_{\text{static lattice (sl)}} + L_{\text{phonon (ph)}}$$

$$\epsilon_{\rm MPB}^{\rm rods,slabs} \approx 4.81 \times 10^{17}\, k_e T_9^8 J \,\mathrm{erg\,s^{-1}\,cm^{-3}}$$

Pethick & Thorsson (1997)

Mixed Phase Neutrino Emissivity



DU: Direct Urca

$$n \rightarrow p + e^- + \bar{\nu}$$

MU: Modified Urca

$$n + p \rightarrow p + p + e^- + \bar{\nu}$$

NPB: Nucleon Pair Braking

MPB: Mixed Phase Bremsstrahlung

$$e + (Z,A) \rightarrow e + (Z,A) + \nu + \bar{\nu}$$

W. Spinella, FW (2017)
Spinella, FW, Orsaria, Contrera (2018)

Rotating Neutron Stars

Friedman, Ipser & Parker (1986)

- Metric:

$$ds^2 = -e^{-2\nu} dt^2 + e^{2(\alpha+\beta)} r^2 \sin^2\theta \, (d\varphi - N^\varphi dt)^2 + e^{2(\alpha-\beta)} (dr^2 + r^2 d\theta^2)$$

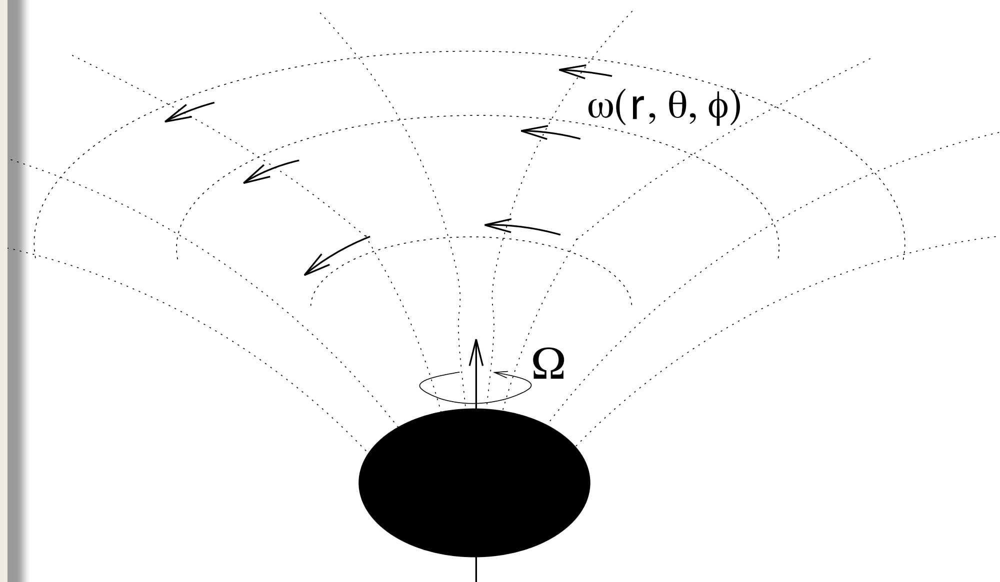
- Frequency range:

$$0 < \Omega < \Omega_K \underset{\text{Newtonian limit}}{\longrightarrow} \sqrt{M/R^3}$$

- Kepler (mass shedding) frequency:

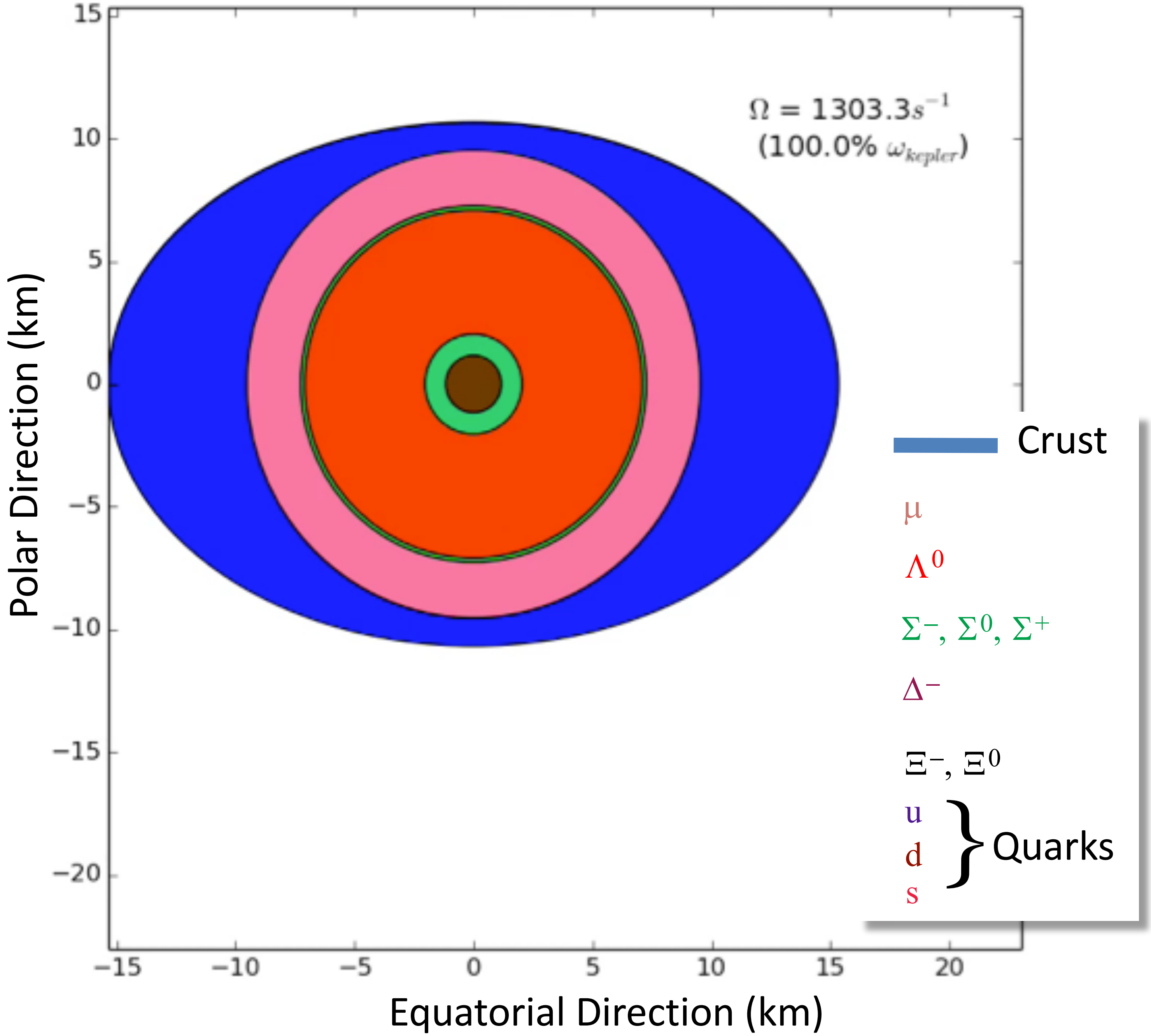
$$\Omega_K = r^{-1} e^{\nu-\alpha-\beta} U_K + N^\varphi$$

- Differential rotation/uniform rotation



➡ Stellar properties: M, R_{pole}, $R_{equator}$, I, z_{pole}, $z_{equator}$, Ω_K

Core Composition of a Rotating Neutron Star



http://fweber.sdsu.edu/~fweber/Mellinger/Compositions.html

Summary

- Gibbs Phase Transition
- Quark-Hadron Lattice
- Blobs, Rods, Plates
- Bremsstrahlung
- Neutrino emissivity (static lattice, phonon scattering)
- Possible Observables: Glitches, Cooling (MXB 1659-29?)