Normal modes of two–superfluid neutron stars with leptonic buoyancy

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Introduction

- Nucleons in neutron star core likely to be superfluids $\rightarrow$ protons and neutrons can move "independently"
- Different mode spectrum, twice as many modes (Lindblom and Mendell 1994, Lee 1995)
- Goal: calculate g–modes and p–modes in spherical, nonrotating, two–superfluid stars, account for electrons and muons. Later add rotation, hyperons, quark matter cores, etc.

Why?

- Low-frequency modes coupling to tides during NS–NS or NS–BH inspiral (Bildsten and Cutler 1992, Lai 1994, etc.) have implications for gravitational wave observations due to GW phase shift- though unlikely for current-generation detectors
- Nonlinear, multi-mode instabilities, if they occur, could have a greater effect on GW phase shift (Essick, Vitale and Weinberg 2016)
- Future: effect of superfluidity, multiple leptonic species on the r–mode instability, which sets upper limit on NS rotation rate. Observed maximum rotation rate (~ 700 Hz) twice the currently-explained theoretical value.

What's different?

- Many calculations of compressional modes in superfluid stars have been performed, some including leptonic buoyancy g-modes (Yu and Weinberg 2017)
- We use a relatively flexible parameterized EOS, allowing us to see how g-mode frequencies vary as functions of mass, nuclear compressibility K and neutron–proton entrainment
- We use a more realistic, two-fluid model of the crust accounting for superfluid neutrons
- We find previously undiscovered nearly-resonant pairs of p-modes

Stellar model and equation of state

- EOS must allow $M \geq 2M_{\odot}$
- Must also include explicit dependence on densities of individual fluid constituents (n, p, e, μ in core, nuclei and dripped neutrons in inner crust)
- Background star calculated using TOV equation
- $T = 0$, no magnetic field, no dissipation, Cowling approximation

Crust

- Use BPS EOS for outer crust, BBP EOS for inner crust
- Ignore outer crust for the purpose of calculating normal modes– set the outer boundary condition ($\Delta P = 0$) at neutron drip
- Inner crust of neutron–rich nuclei surrounded by electron gas and dripped neutron (super)fluid. Neutron drip begins at $\rho_m \sim 3 \times 10^{11}$ g/cm^3, nuclei fully dissolve at $\rho_m \sim 1 \times 10^{14}$ g/cm^3
- At densities just above neutron drip but within inner crust, neutron pairing gap goes to 0 even at $T = 0$ (Gezerlis, Pethick and Schwenk 2014) $\rightarrow$ small (~ 10 m) layer of single fluid in inner crust where normal fluid nuclei are comoving with nuclei

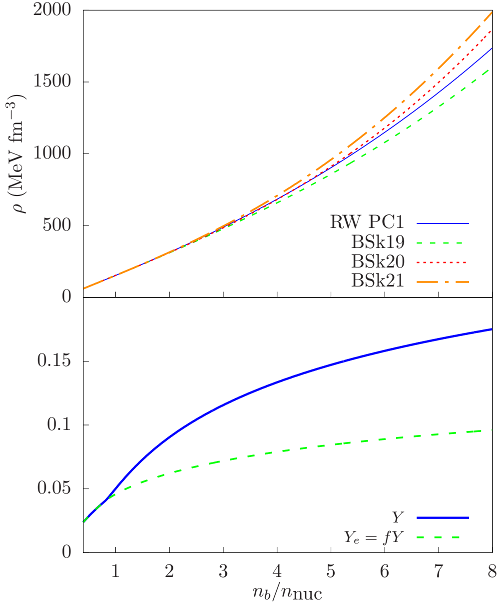
(Outer) Core Equation of state

- $\rho = \rho(n_b, Y = n_p/n_b, f = n_e/n_p)$, beta equilibrium gives $f = f(n_b, Y)$. Muons present for $n_b > 0.8 n_{\mathsf{nuc}}$.
- npeμ gas kinetic energy plus simple nuclear interaction based on Hebeler+ 2013

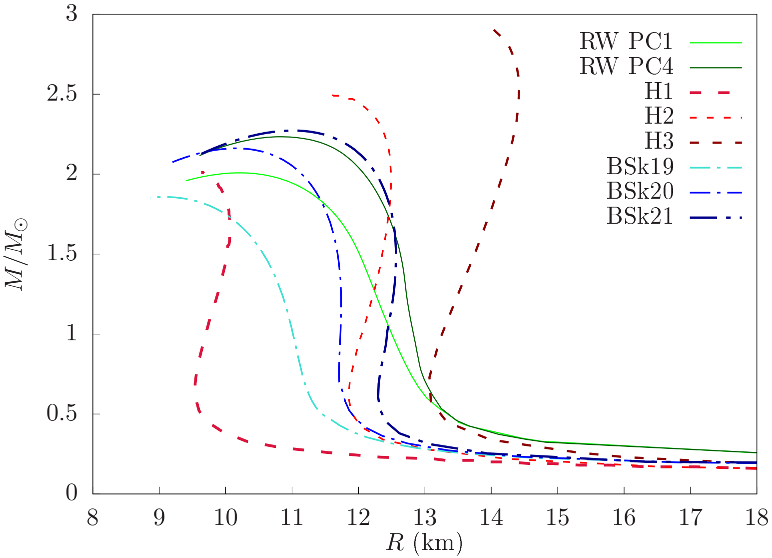
$$\rho_{\mathsf{int}}(n_b, Y) = n_{\mathsf{nuc}} E_S \frac{\overline{n}^2 + f_S \overline{n}^{\gamma_S + 1}}{1 + f_S} + n_{\mathsf{nuc}} E_A \overline{n}^2 \left(\frac{\overline{n} + \overline{n}_0}{1 + \overline{n}_0} \right)^{\gamma_A - 1} (1 - 2Y)^2,$$

- Modified to give energy per baryon $\propto n_b$ at low densities, since used same interaction term for nuclear energy in BBP EOS

- Parameters chosen to allow $2M_\odot$ neutron star, satisfy nuclear physics constraints: $E_{\text{binding}} = -16$ MeV, zero pressure, reasonable S_v and L for symmetric nuclear matter at saturation density
- $E_S = -37.8$ MeV, $E_A = 19.9$ MeV, $\gamma_A = 0.61$, $\overline{n}_0 = 0.05$ fixed– these had only a small effect on the maximum mass
- $S_v = 31.7$ MeV, $L = 60.3$ MeV
- $-0.667 < f_S < -0.530$, $1.31 < \gamma_S < 1.547$ to vary nuclear compressibility $230 < K < 280$ MeV; label parameterizations by value of K



2000
1500
1000
500
0
ρ (MeV fm^{-3})
RW PC1
BSk19
BSk20
BSk21
0.15
0.1
0.05
0
Y
$Y_e = fY$
1
2
3
4
5
6
7
8
$n_b/n_{\rm nuc}$



RW PC1
RW PC4
H1
H2
H3
BSk19
BSk20
BSk21
$M/M_\odot$
R (km)
0
0.5
1
1.5
2
2.5
3
8
9
10
11
12
13
14
15
16
17
18

Two–superfluid formalism

- Since $\omega_{\text{plasma}} \gg \omega_{p,g}$, leptons co–move with the protons
- Describe perturbations with two displacement fields: In the core, $\boldsymbol{\xi}_n$ (neutrons) and $\boldsymbol{\xi}_q$ ("charged", protons plus leptons)
- In the inner crust, two fluids are $\boldsymbol{\xi}_c$ (nuclei) and $\boldsymbol{\xi}_f$ (dripped neutron superfluid). Use single displacement field for small single-fluid region above neutron drip
- For non–rotating, spherically–symmetric stars, can take as local displacement field

$$\boldsymbol{\xi}_a = e^{i\omega t}\left[\xi_a^r(r) Y_{lm}\hat{\mathbf{e}}_r + \xi_a^{\perp}(r) r \nabla Y_{lm}\right]$$

- Perturbing the Euler equations gives relations between displacement fields and chemical potential perturbations. In the core

$$0 = e^{-\nu}(1-\epsilon_n)\partial_t^2\xi_n^r + e^{-\nu}\epsilon_n\partial_t^2\xi_q^r + e^{-\lambda/2}\frac{d}{dr}\left(\frac{\delta\mu_n}{\mu_0}\right)$$

$$0 = e^{-\nu}(1-\epsilon_n)\partial_t^2\xi_n^\perp + e^{-\nu}\epsilon_n\partial_t^2\xi_q^\perp + \frac{1}{r}\left(\frac{\delta\mu_q}{\mu_0}\right)$$

$$0 = e^{-\nu}(1-\epsilon_p)\partial_t^2\xi_q^r + e^{-\nu}\epsilon_p\partial_t^2\xi_n^r + e^{-\lambda/2}\frac{d}{dr}\left(\frac{\delta\mu_q}{\mu_0}\right) + \frac{\delta\mu_\mu - \delta\mu_e}{\mu_0}\frac{df_0}{dr}$$

$$0 = e^{-\nu}(1-\epsilon_p)\partial_t^2\xi_q^\perp + e^{-\nu}\epsilon_p\partial_t^2\xi_n^\perp + \frac{1}{r}\left(\frac{\delta\mu_q}{\mu_0}\right)$$

df_0/dr = lepton composition gradient, $e^{\nu(r)} = -g_{tt}$, $e^{\lambda(r)} = g_{rr}$

- Two fluids are coupled *thermodynamically* through $\delta\mu_n$ and $\delta\mu_q$, in addition to coupling via entrainment
- Entrainment in core based on Prix and Rieutord 2002 model:

$$n_q \epsilon_p = n_n \epsilon_n$$
$$\epsilon_p = 1 - \frac{m_p^*}{m_N}$$

- Varied ϵ_p between 0 and 0.5

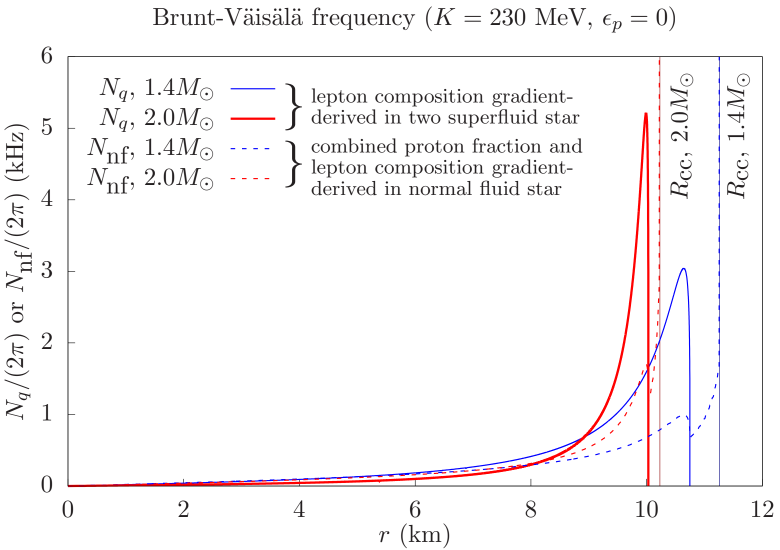
Leptonic Brunt-Väisälä frequency

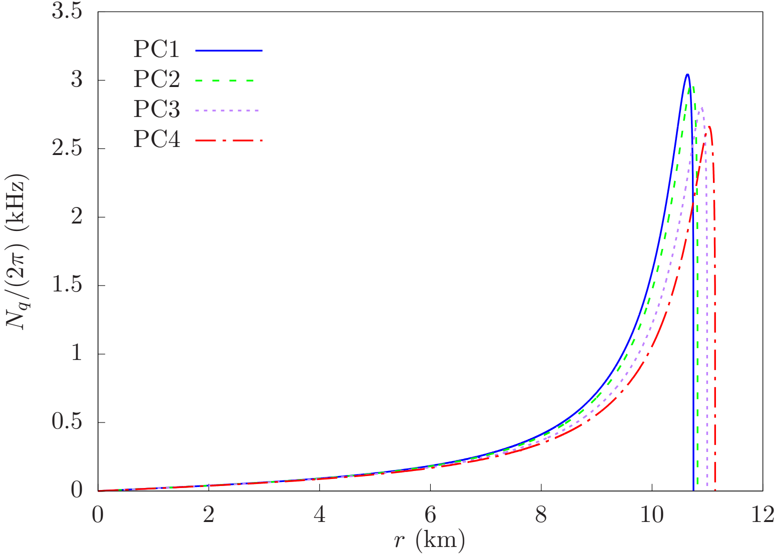
- Obtain buoyancy due to df_0/dr- perturbed fluid no longer in beta equilibrium

$$N_q^2(r) = -e^{\nu-\lambda}\frac{\epsilon_p}{(1-\epsilon_p-\epsilon_n)}\left(\frac{\mu_{qf}}{\mu_0}\frac{d\mu_0}{dr}\right)\left(\frac{\mu_{nn}-\mu_{nq}}{n_q(\mu_{nn}\mu_{qq}-\mu_{nq}^2)}\right)\frac{df_0}{dr}$$

where $\mu_{xy} = \frac{\partial \mu_x}{\partial n_y}$

- $N_q^2 = 0$ below muon threshold density
- First found by Gusakov and Kantor 2014


Brunt-Väisälä frequency ($K = 230$ MeV, $\epsilon_p = 0$)
N_q, $1.4M_\odot$
N_q, $2.0M_\odot$
lepton composition gradient-derived in two superfluid star
$N_{\rm nf}$, $1.4M_\odot$
$N_{\rm nf}$, $2.0M_\odot$
combined proton fraction and lepton composition gradient-derived in normal fluid star
$R_{\rm cc}$, $2.0M_\odot$
$R_{\rm cc}$, $1.4M_\odot$
$N_q/(2\pi)$ or $N_{\rm nf}/(2\pi)$ (kHz)
r (km)
0
1
2
3
4
5
6
0
2
4
6
8
10
12


PC1
PC2
PC3
PC4
3.5
3
2.5
2
1.5
1
0.5
0
0
2
4
6
8
10
12
$N_q/(2\pi)$ (kHz)
r (km)

Calculating the modes

- Defining $\Pi_a \equiv \frac{\delta\mu_a}{\mu_0}$ and using the perturbed continuity equation, we obtain a system of four coupled first–order ODEs

$$\frac{d\xi_n^r}{dr} + \left[\frac{2}{r} + \frac{d\ln n_n}{dr}\right]\xi_n^r + \left[-\frac{k_\perp^2}{\omega^2}e^\nu + \frac{\mu_0\mu_{qq}}{n_n D}\right]e^{\lambda/2}\Pi_n = \frac{\mu_{nq}\mu_0}{n_n D}e^{\lambda/2}\Pi_q + \frac{\mu_{nq}\mu_{qf}}{n_n D}\frac{df_0}{dr}\xi_q^r$$

$$\frac{d\xi_q^r}{dr} + \left[\frac{2}{r} + \frac{d\ln n_q}{dr} + \frac{\mu_{nn}\mu_{qf}}{n_q D}\frac{df_0}{dr}\right]\xi_q^r + \left[-\frac{k_\perp^2}{\omega^2}e^\nu + \frac{\mu_0\mu_{nn}}{n_q D}\right]e^{\lambda/2}\Pi_q = \frac{\mu_0\mu_{nq}}{n_q D}e^{\lambda/2}\Pi_n$$

$$\xi_n^r\omega^2 = e^{\nu-\lambda/2}\frac{d\Pi_n}{dr}$$

$$\xi_q^r(\omega^2 - N_q^2) = e^{\nu-\lambda/2}\frac{d\Pi_q}{dr} + \frac{\mu_0 N_q^2 e^{\lambda/2}(\mu_{nn}\Pi_q - \mu_{nq}\Pi_n)}{(d\mu_0/dr)(\mu_{nn} - \mu_{nq})}$$

where $\mu_{qf} = \frac{\partial\mu_q}{\partial f}$, $D = \mu_{nn}\mu_{qq} - \mu_{nq}^2$, $k_\perp^2 = \frac{l(l+1)}{r^2}$ (No entrainment shown here)

- $\mu_{nq} = \frac{\partial \mu_n}{\partial n_q} = \mu_{qn}$ responsible for thermodynamic coupling of fluids
- Similar set of equations for two-fluid crust, but $n \to f$, $q \to c$, $N_q = 0$
- μ_{xy} for crust complicated: No beta equilibrium in perturbed fluid elements, but impose chemical, mechanical equilibrium and the "nuclear virial theorem"
- Perturb ρ, conditions for mechanical, chemical equilibrium $\to$ obtain $\mu_c(n_c, n_f)$, $\mu_f(n_c, n_f)$

Boundary/Interface conditions

- Regularity at $r = 0$, $\Delta P = 0$ at surface
- Crust–Core interface:

$$\xi^r_q|_+ = \xi^r_c|_- \quad \text{(interface moves with charged fluid)}$$
$$(n_n \xi^r_n - n_n \xi^r_q)_+ = (n_f \xi^r_f - n_f \xi^r_c)_- \quad \text{(baryon conservation)}$$
$$\Delta_c P|_+ = \Delta_c P|_- \quad \text{(continuity of traction)}$$
$$\delta\mu_n|_+ = \delta\mu_f|_- \quad \text{(“chemical gauge” independence)}$$

- Two-fluid–single fluid interface:

$$(n_f \xi^r_f + n_c \xi^r_c)_+ = (n_b \xi^r_b)_- \quad \text{(baryon conservation)}$$
$$(n_f \delta\mu_f + n_c \delta\mu_c)_+ = (n_b \delta\mu_b)_- \quad \text{(continuity of traction)}$$
$$\Delta_f n_f|_{r=R_{\mathsf{SFT}}} = 0 \quad \text{(Phase transition moves with SF neutrons)}$$

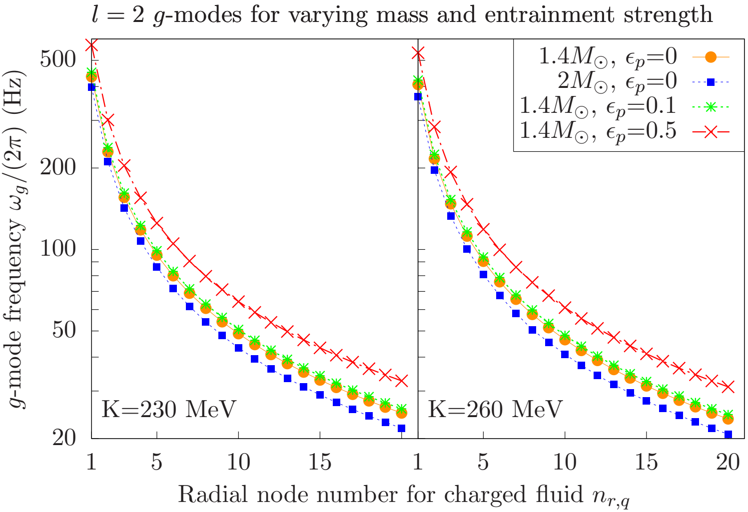
g–modes

- Low–frequency buoyancy modes arising from composition or temperature gradients
- Approximate dispersion relation

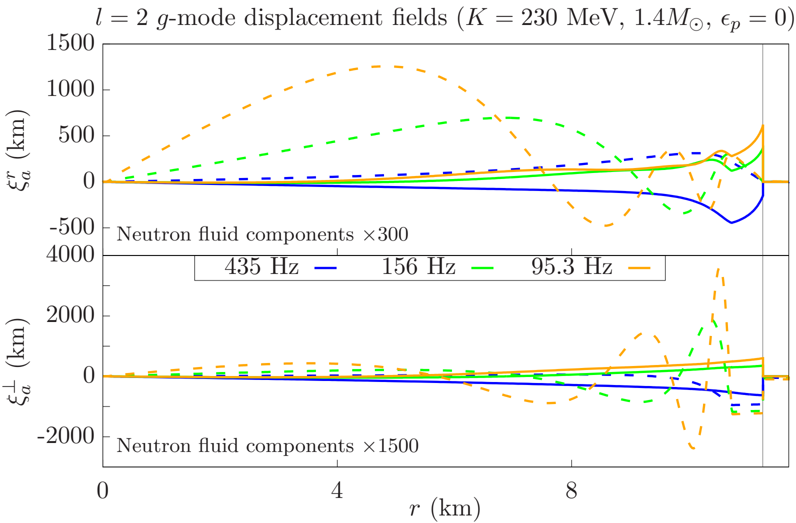
$$\omega_g^2 = N_q^2 \frac{k_\perp^2 e^\lambda}{k_r^2 + k_\perp^2 e^\lambda}$$

- Allowed frequencies are roughly set by (WKB approximation)

$$n_r \pi = \int_{r_{\text{in}}}^{r_{\text{out}}} k_r dr \rightarrow \omega_g \propto n_r^{-1}$$



$l = 2$ g-modes for varying mass and entrainment strength
$1.4M_\odot$, ϵ_p=0
$2M_\odot$, ϵ_p=0
$1.4M_\odot$, ϵ_p=0.1
$1.4M_\odot$, ϵ_p=0.5
K=230 MeV
K=260 MeV
g-mode frequency $\omega_g/(2\pi)$ (Hz)
Radial node number for charged fluid $n_{r,q}$



Solid/dashed lines used for neutron/charged fluid

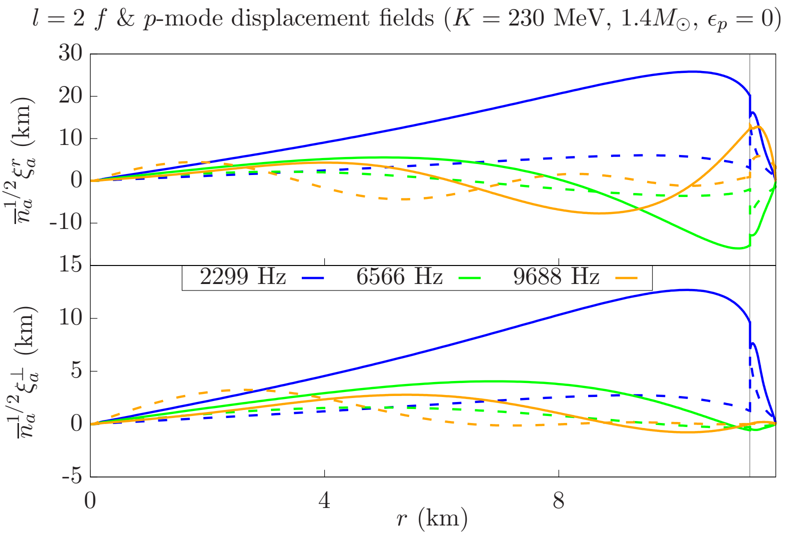
- WKB approximation is accurate to within ≤ 2 % for $n_{r,q} > 2$
- Approximate g-mode dispersion as function of mass, nuclear compressibility, entrainment (≤ 5 % deviation for $n_{r,q} > 2$):

$$\frac{\omega_g}{2\pi} \approx \frac{608 - 0.83(K - 240 \text{ MeV}) - 90\frac{M}{M_\odot} + 297\epsilon_p}{n_{r,q}} \text{ Hz}, \quad (1)$$

p–modes

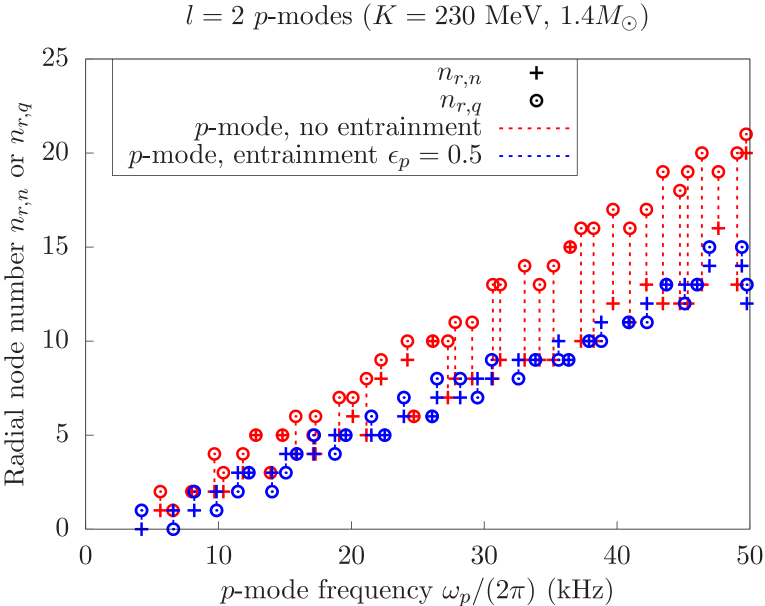
- High–frequency acoustic modes $\omega_p \propto n_r$
- Naive approximate dispersion relation in core: $\omega_p^2 = c_{s\pm}^2 k^2$ where

$$c_{s\pm}^2 = e^{\nu-\lambda}\frac{n_n n_q}{2\mu_0}\left[\left(\frac{\mu_{qq}}{n_n}+\frac{\mu_{nn}}{n_q}\right) \pm \sqrt{\left(\frac{\mu_{qq}}{n_n}-\frac{\mu_{nn}}{n_q}\right)^2+\frac{4\mu_{nq}^2}{n_n n_q}}\right]$$



Solid/dashed lines used for neutron/charged fluid

- Two sets of p–modes corresponding to two superfluids
- n, q fluid contributions to each p–mode do not have same radial node number, are nearly uncoupled (Kantor and Gusakov 2011, Gualtieri+ 2014)
- Pairing between uncoupled n, q modes with similar frequency occurs, which shifts the combined mode frequency away from the uncoupled mode frequencies and can result in creation of nearly-resonant p-mode pairs with $\Delta\omega_p \sim \omega_g$- could be a source of nonlinear three–mode instabilities (Weinberg, Arras and Burkart 2013, Weinberg 2016)
- Strong entrainment forces both fluids to move together with “neutron-dominated” dispersion



$l = 2$ p-modes ($K = 230$ MeV, $1.4M_\odot$)
Radial node number $n_{r,n}$ or $n_{r,q}$
p-mode frequency $\omega_p/(2\pi)$ (kHz)
$n_{r,n}$
$n_{r,q}$
p-mode, no entrainment
p-mode, entrainment $\epsilon_p = 0.5$
0
5
10
15
20
25
0
10
20
30
40
50

Conclusion

- Reproduced previous calculations of g- and p-modes in two-fluid neutron stars with leptonic buoyancy, but included more realistic two-fluid crust
- Two-fluid crust boundary conditions remove radial nodes from n fluid g-mode displacement fields
- Approximate expression for g-mode frequencies as function of M, K and entrainment- should check for different equations of state
- Find closely–spaced p–mode frequencies $\rightarrow$ potential for large nonlinear couplings between tide and two p–modes, though exact details of this in two–fluid case aren't known