Towards accurate description of non-equilibrium dynamics in superfluid neutron matter

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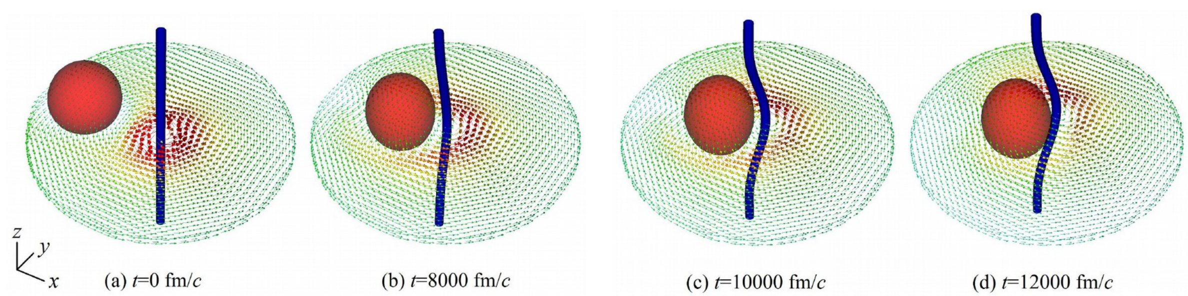
University of Washington



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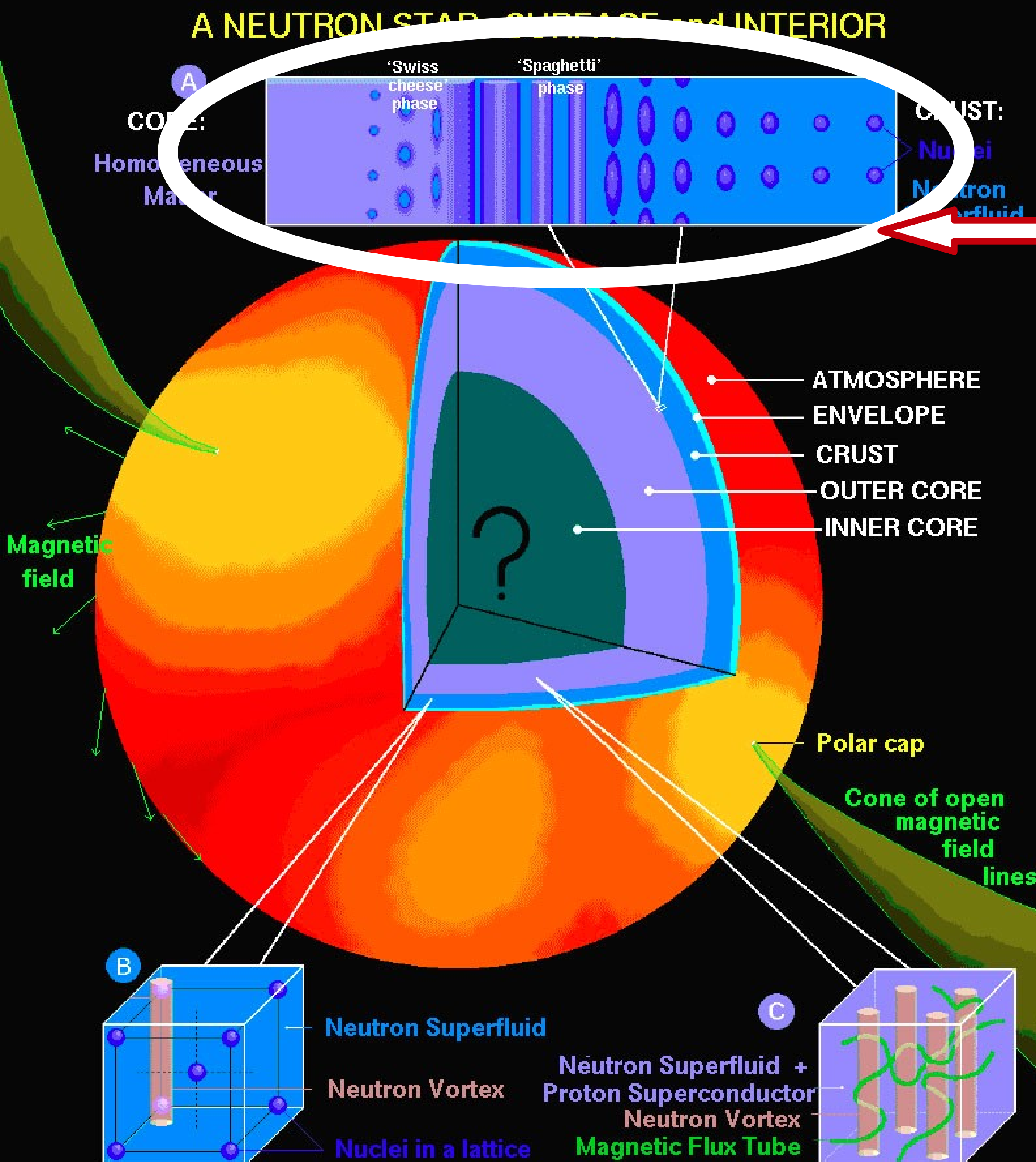
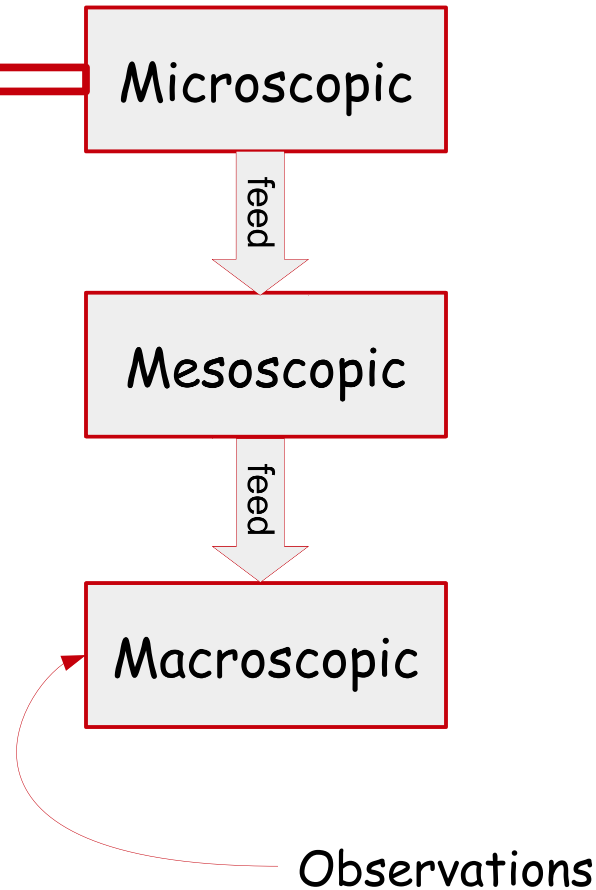
CSQCD VII, New York, June 14, 2018

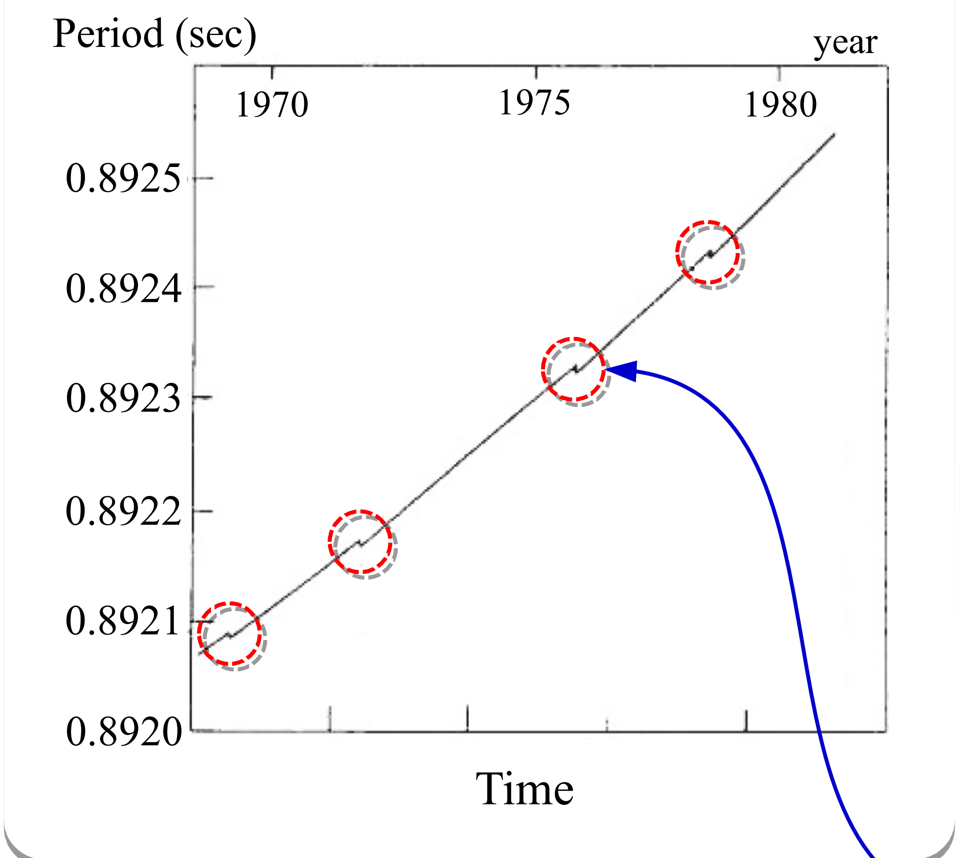
Hierarchy of theories:





Our motivation: Glitch
(a sudden increase of the rotational frequency)

- Glitches in the Vela pulsar



V.B. Bhatia, A Textbook of Astronomy and Astrophysics with Elements of Cosmology, Alpha Science, 2001.

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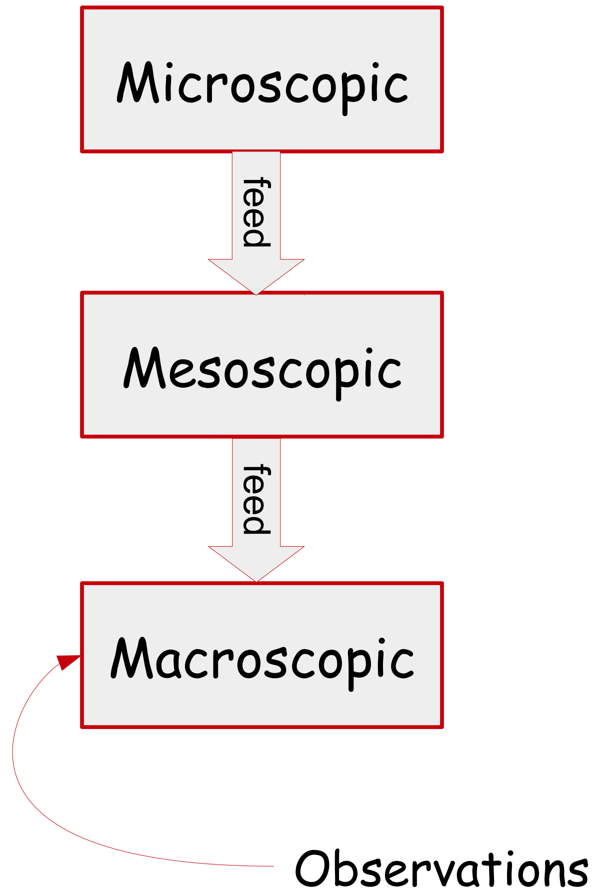


- **Vortex model**
  (P. W. Anderson and N. Itoh, Nature 256 (1975))
  - Presently the standard picture for pulsar glitches

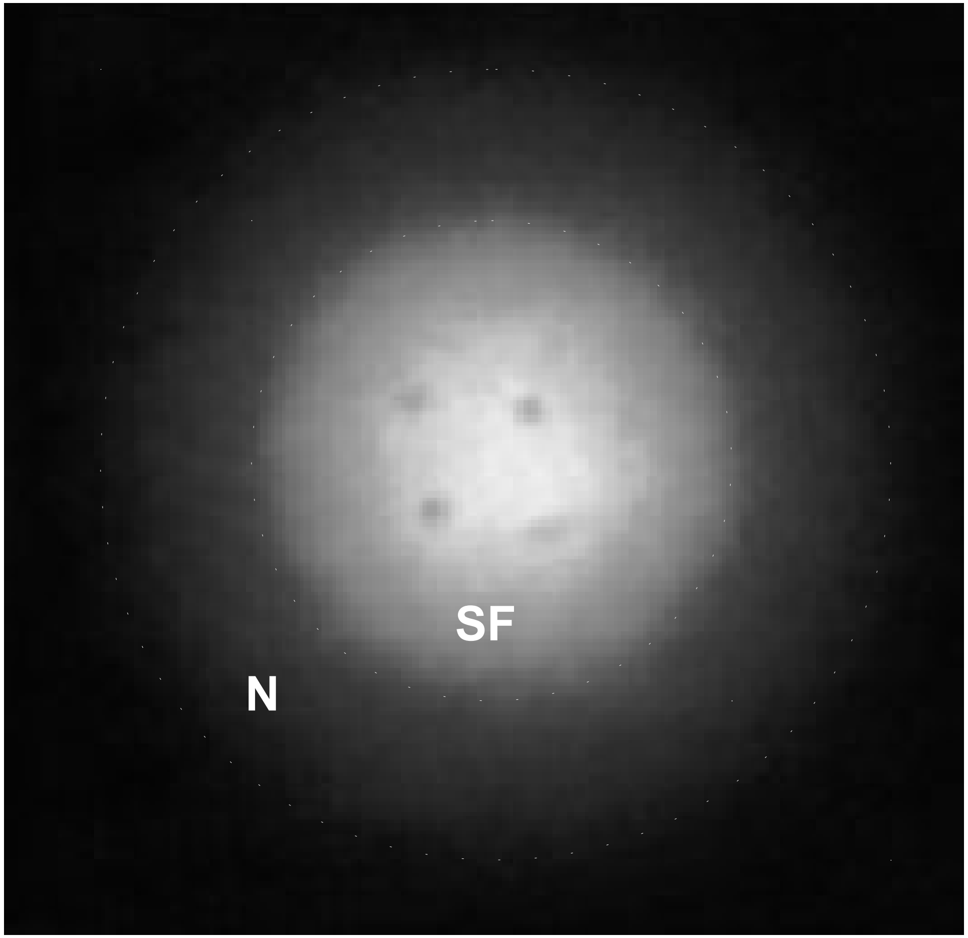
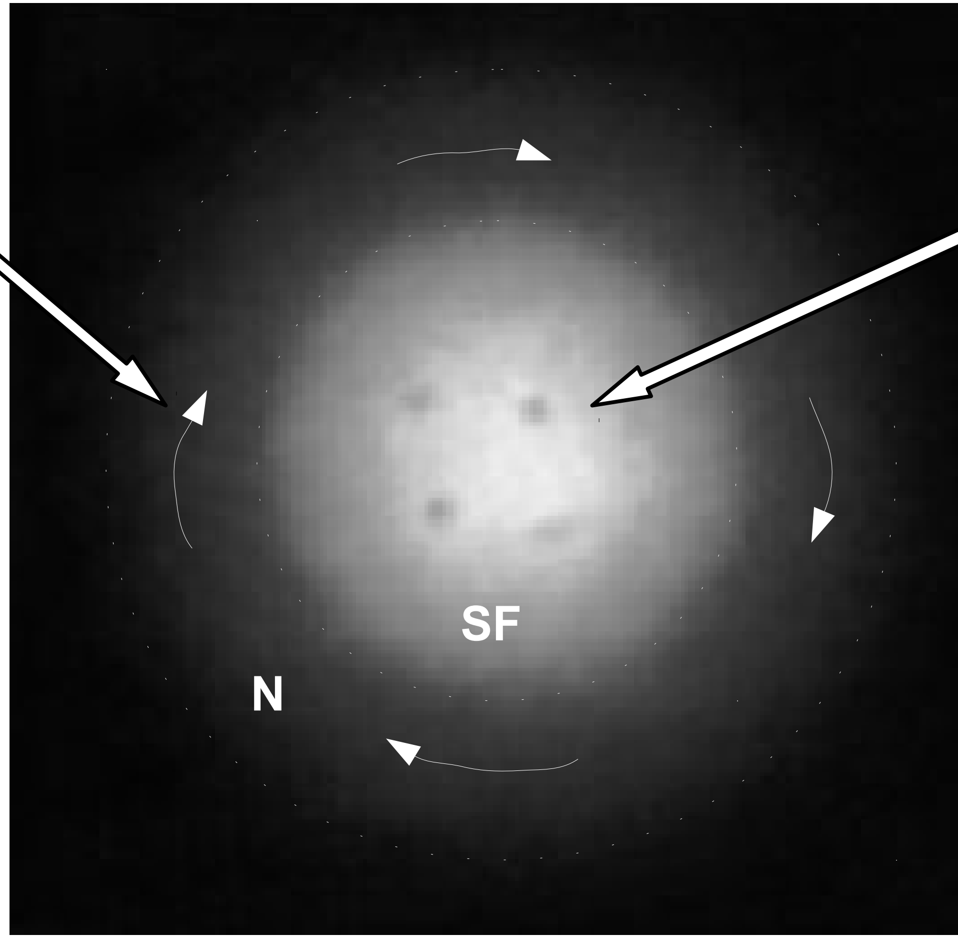


Figure taken from: Zwierlein, et. al, Science 311, 492 (2006)

Normal component
- rigid body rotation

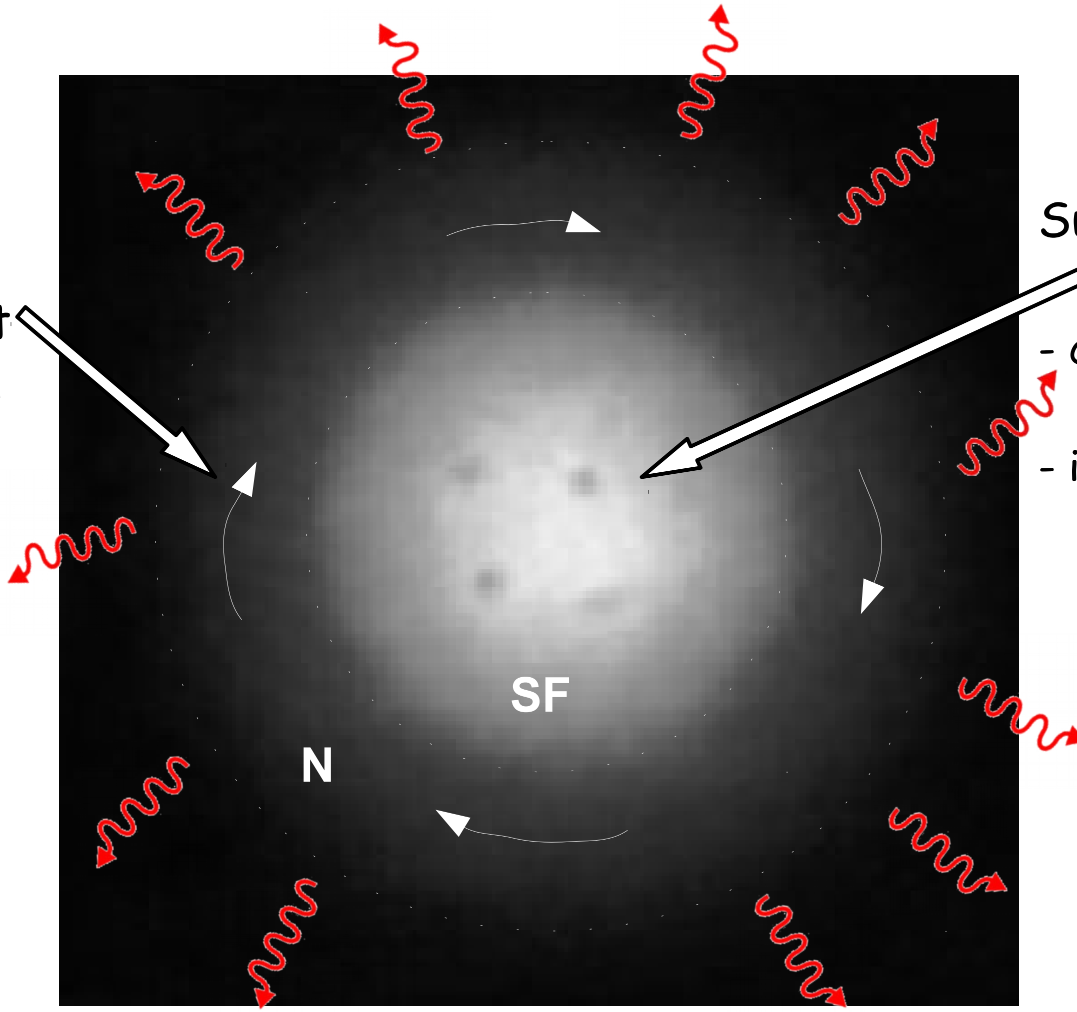


Superfluid component
- can rotate only in form of vortices

Figure taken from: Zwierlein, et. al, Science 311, 492 (2006)

Normal component
- rigid body rotation
- slows down due to energy radiation

Superfluid component
- can rotate only in form of vortices
- in order to decrease the angular momentum number of vortices must change



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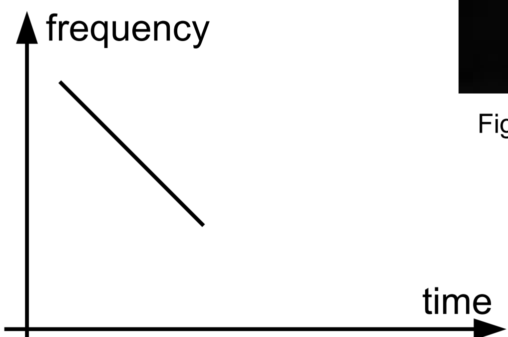

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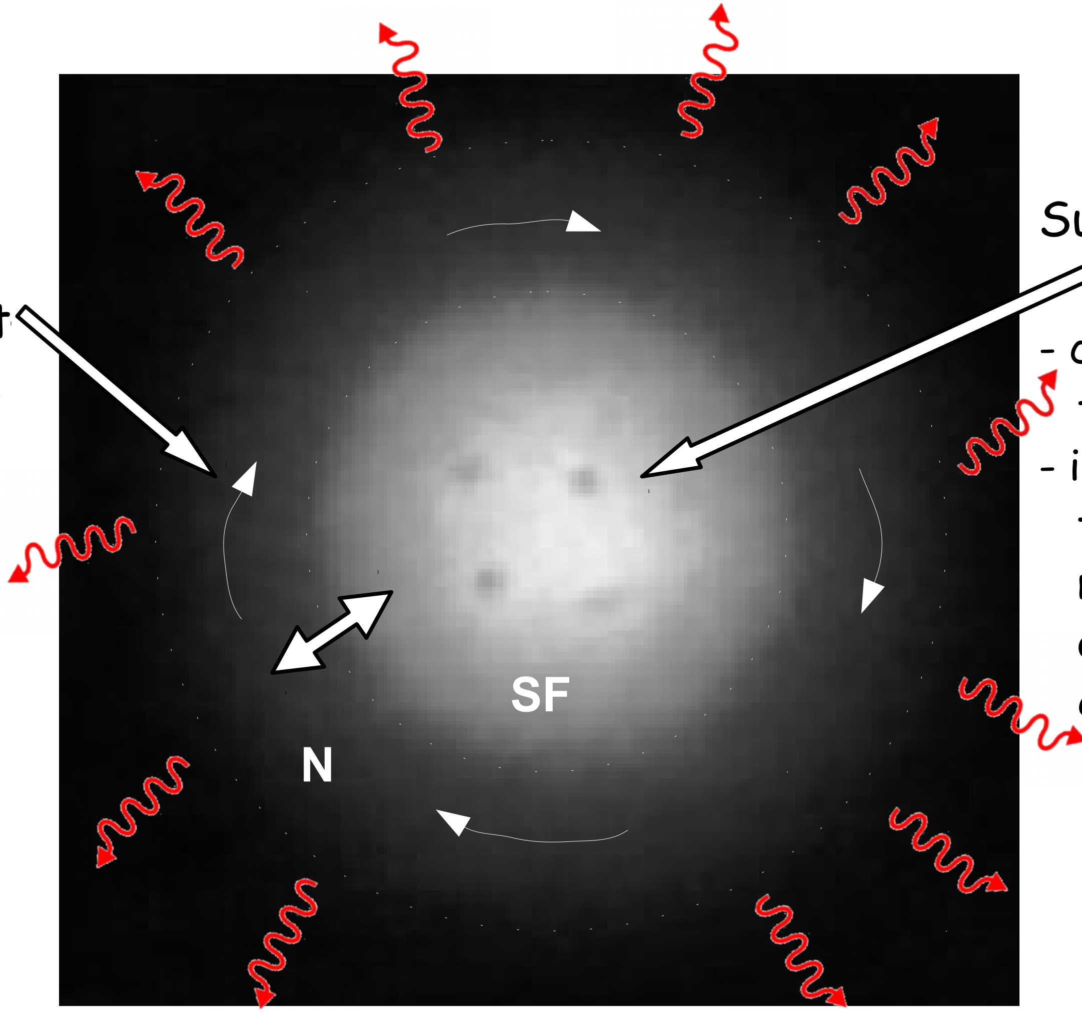

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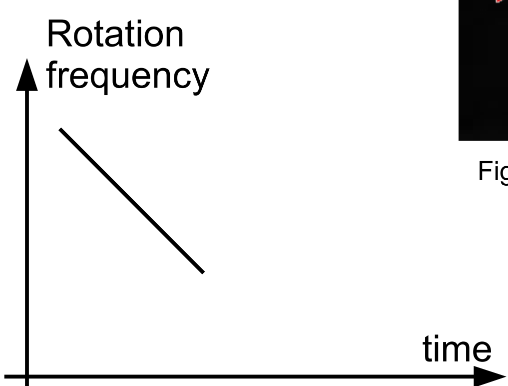
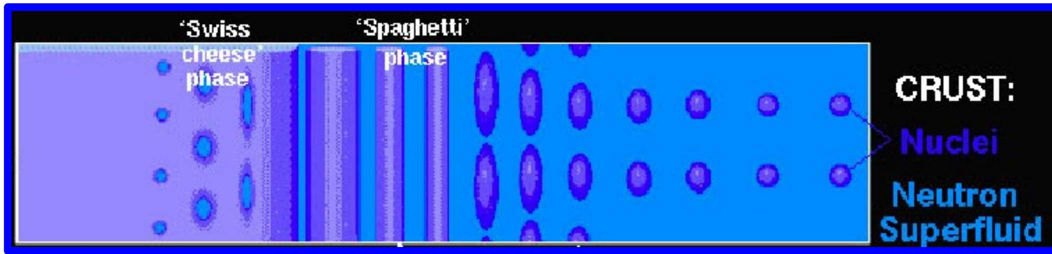
Tension between N and SF component is generated!





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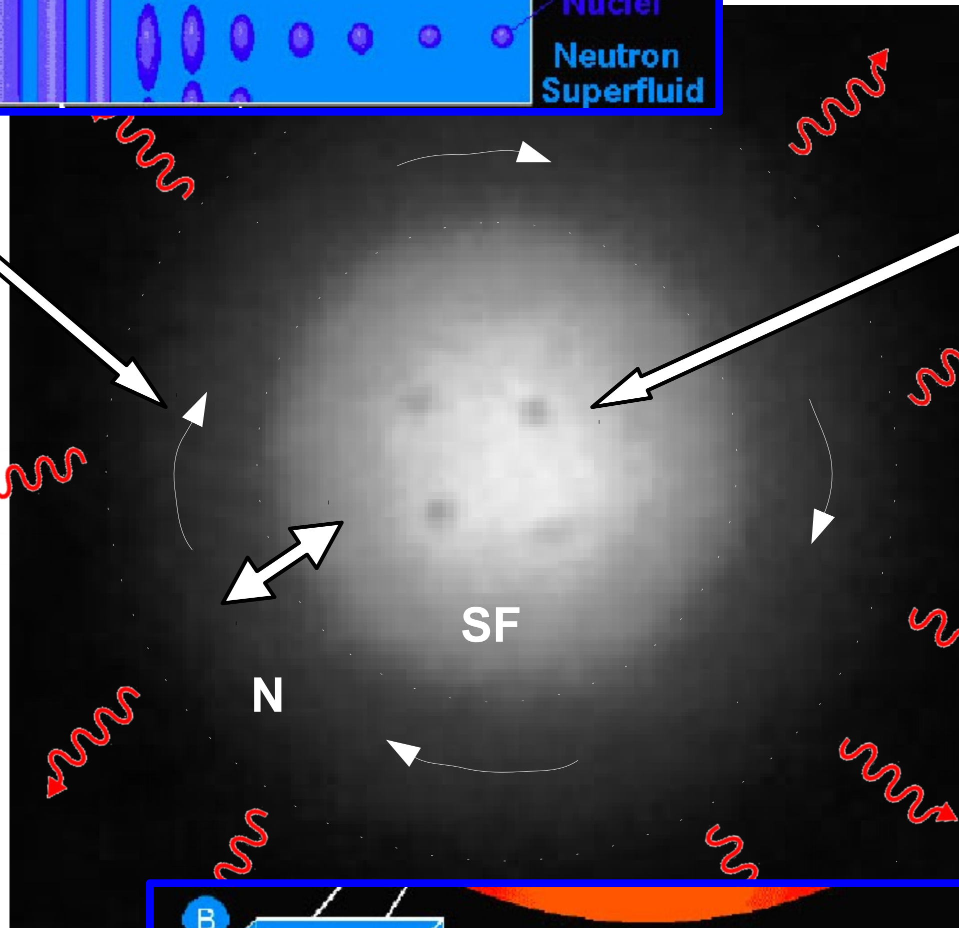

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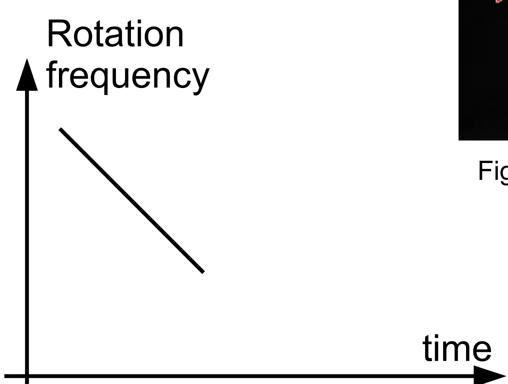
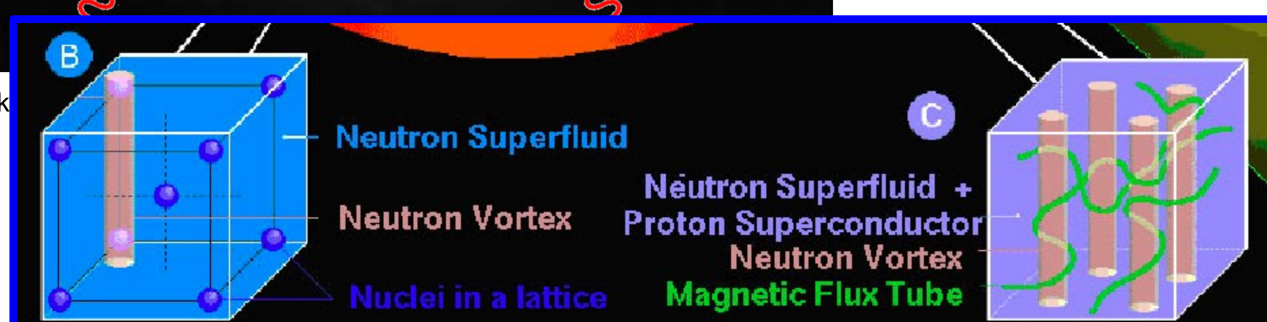


Figure tak



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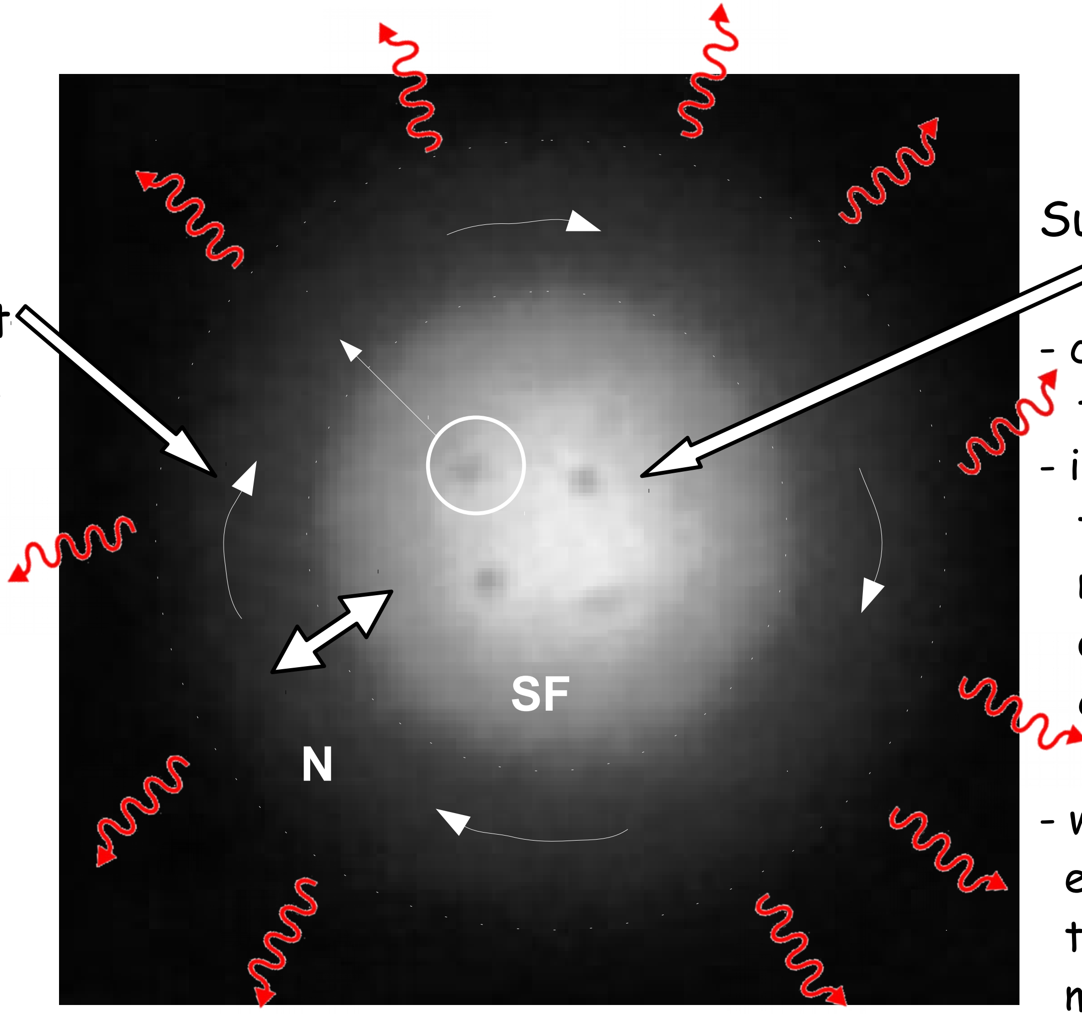
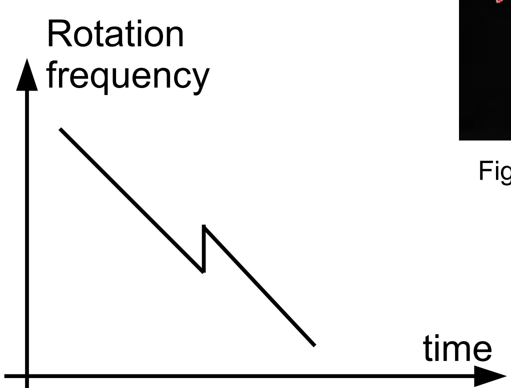


Figure taken from: Zwierlein, et. al, Science 311, 492 (2006)

Superfluid component
- can rotate only in form of vortices
- in order to decrease the angular momentum number of vortices must change

Tension between N and SF component is generated!

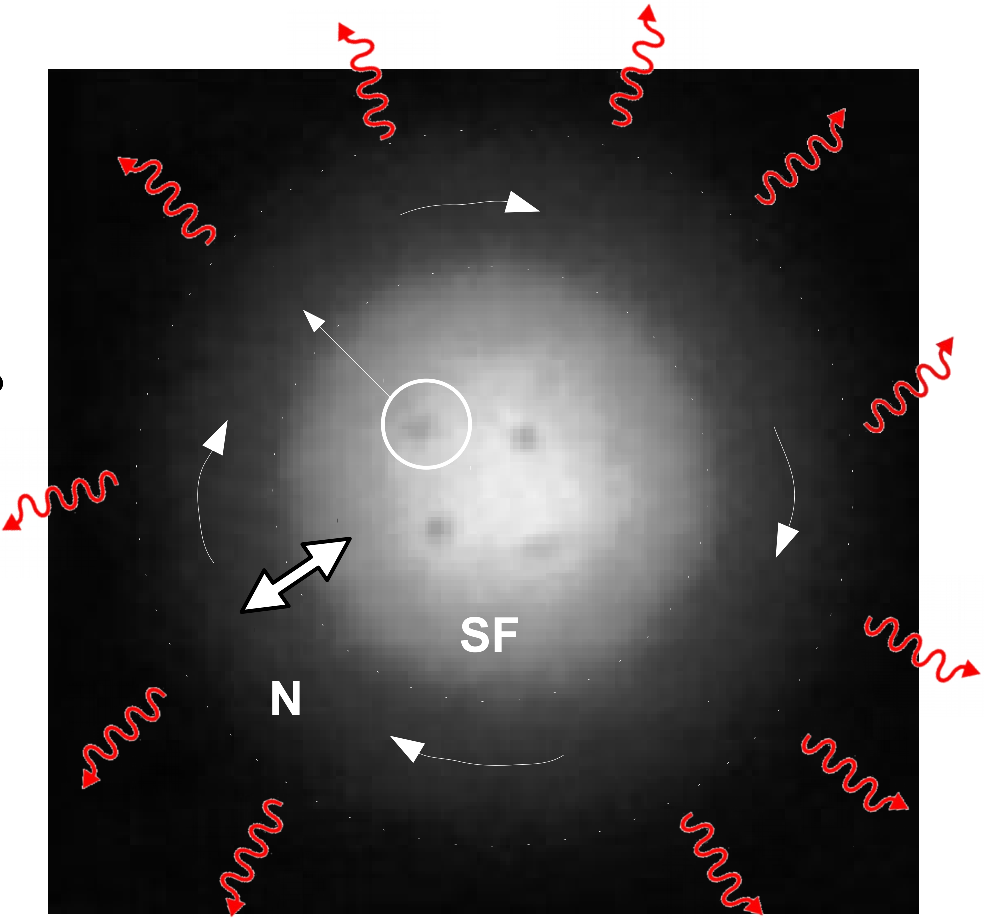
- when vortices are ejected they transfer its angular momentum to N component

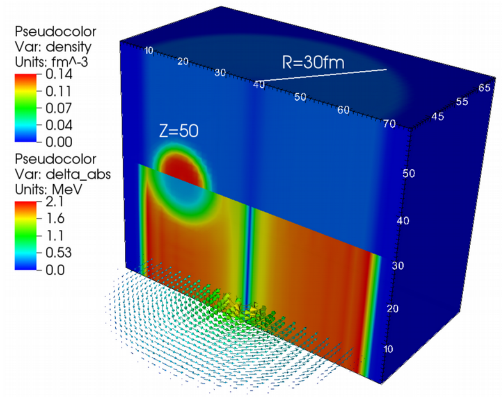


GLITCH!

A lot of open problems:

- …
- …
- origin of pinning mechanism?
- vortex avalanche trigger mechanism?
- regular lattice of vortices
  or tangle of vortices?
- …
- …





Method: TDDFT
DoF: fermionic
(neutrons, protons...)

Hierarchy of theories:

Microscopic

feed

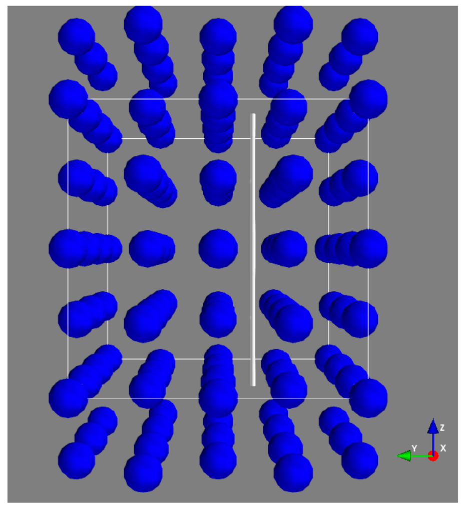
Mesoscopic

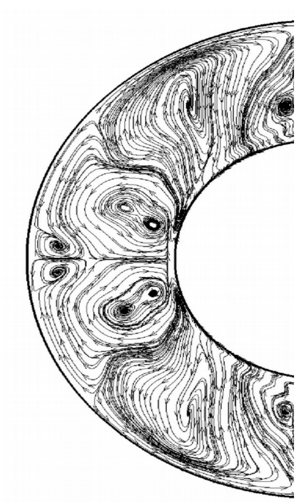
feed

Macroscopic

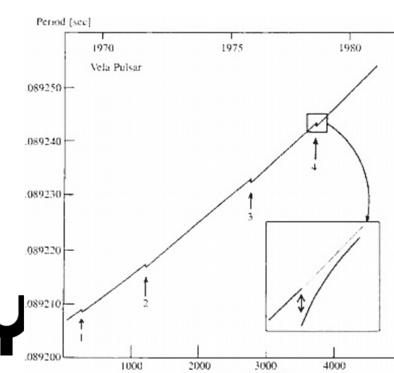
Observations

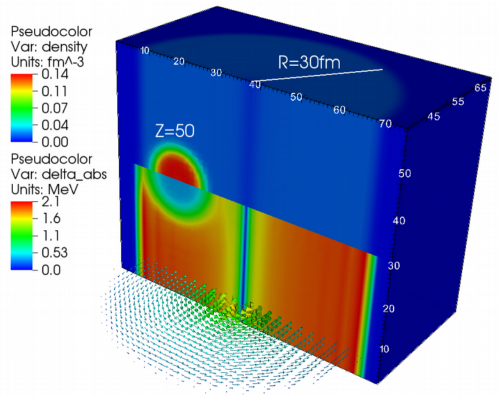
Method: Vortex filament model
DoF: impurities and vortices





Method: Hydrodynamics
DoF: fluid elements





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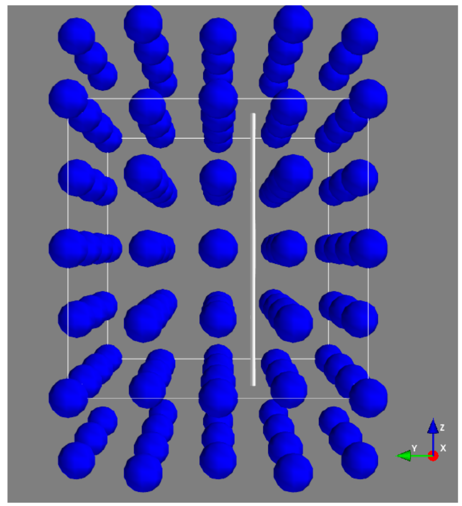

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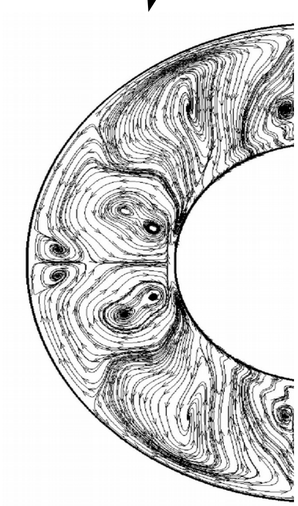
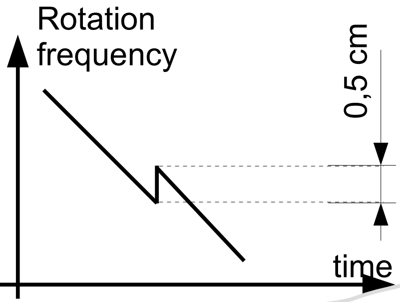


Method: Hydrodynamics
DoF: fluid elements

Matching theories is hard..

- ...
- scaling problem...
- output → input conversion
- ...
- collective effort...
- ...

PHAROS
THE MULTI-MESSENGER PHYSICS AND ASTROPHYSICS OF NEUTRON STARS
http://www.pharos.ice.csic.es/
(cost action CA16214)



Observed relative sizes of glitches:

$$\Delta\nu/\nu \sim 10^{-11} - 10^{-5}$$

maximum glitch amplitude

$$\Delta\Omega_{\max} = \frac{\pi^2}{\kappa I}\int_0^{R_d} dr\, r^3\, f_P(r)$$

quantum of circulation

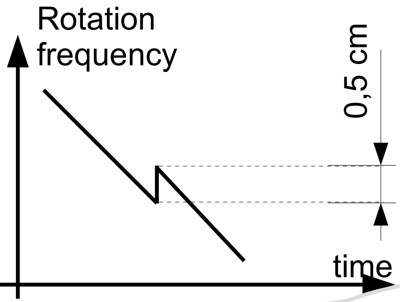
moment of inertia

pinning force

(force needed to move a vortex through the medium with “impurities”)

Result is weakly sensitive to various assumptions of hydro model...

- P. Pizzochero, M. Antonelli, B. Haskell, S. Seveso, Nature Astronomy 1, 0134 (2017)
- M. Antonelli, P. Pizzochero, Journal of Physics: Conf. Series 861 (2017) 012024
- M. Antonelli, A. Montoli, P. M. Pizzochero, MNRAS 475, 5403 (2018)



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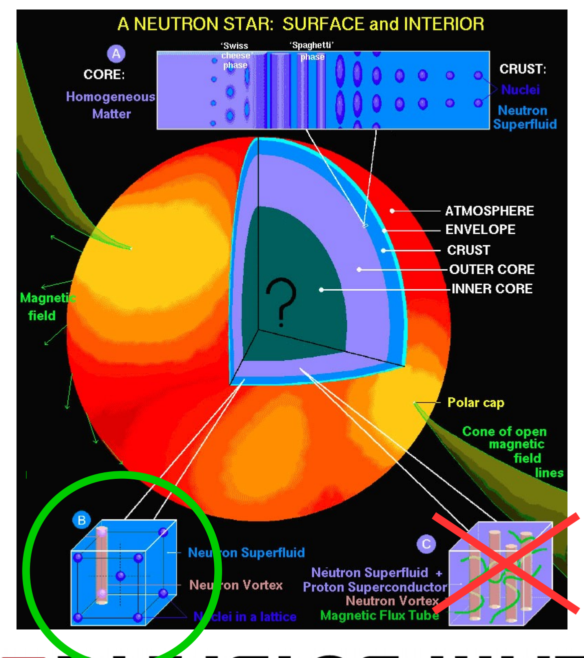
$\Delta\nu/\nu \sim 10^{-11} - 10^{-5}$

$$\Delta\Omega_{\max} = \frac{\pi^2}{\kappa I}\int_0^{R_d} dr\, r^3 f_P(r)$$

Observation

Theory

Assumption: only crust contributes



… now also quantity matters
(not only quality) … → ...
… reliable theory needed!

Challenge for MBT:

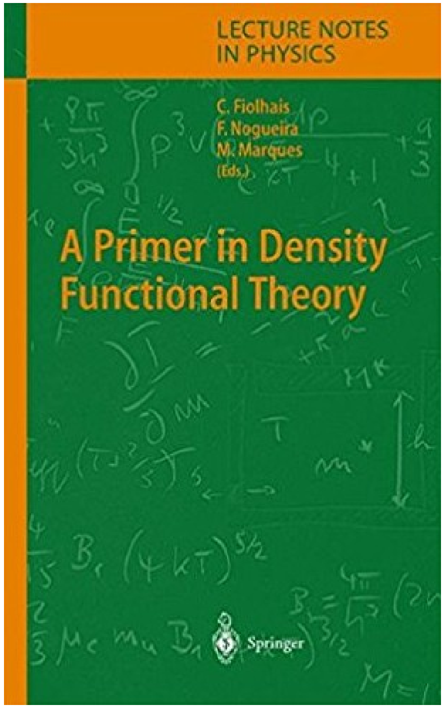
Qualitatively and **quantitatively** accurate

Unified description of static and dynamic properties of **large Fermi systems**

$$i\hbar\frac{\partial}{\partial t}\psi = \hat{H}\psi$$

We know what Eq. should be solved...
The only problem:
How to do it in practice?

Input:
energy density functional



Methods:

- QMC (static)
- *DFT* (static and *dynamic*)
- ...

feed

Alternative frameworks

Schrödinger

$$i\hbar\frac{\partial\Psi}{\partial t} = (\hat{H}_{\text{int}} + \hat{U}_{\text{ext}})\Psi$$

$$\hat{H}_{\text{int}} = \hat{T} + \hat{V}$$

- Derivation of H_{int} - "*easy*"
- Solving many body Schrödinger equation - "hard"

TDDFT

$$E[n] = E_{\text{int}}[n] + \int n(\boldsymbol{r},t)U_{\text{ext}}(\boldsymbol{r},t)\,d\boldsymbol{r}$$

$$E_{\text{int}}[n] = T[n] + V[n] + \ldots$$

- Derivation of E_{int} - "hard"
- Solving emerging equations of motion equation - "*easy*"

Alternative frameworks

Schrödinger

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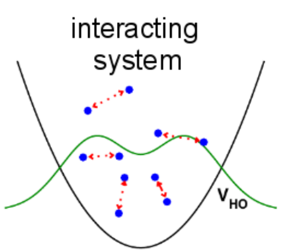
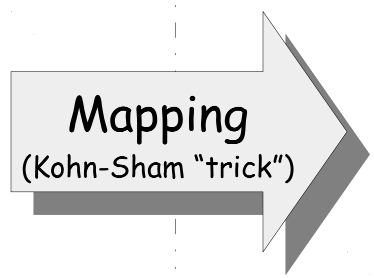
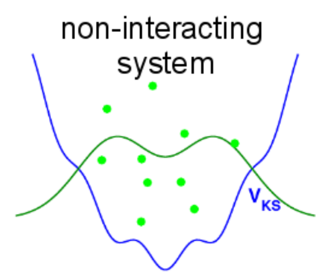
$$|\Psi\rangle$$





$$|\Phi\rangle$$

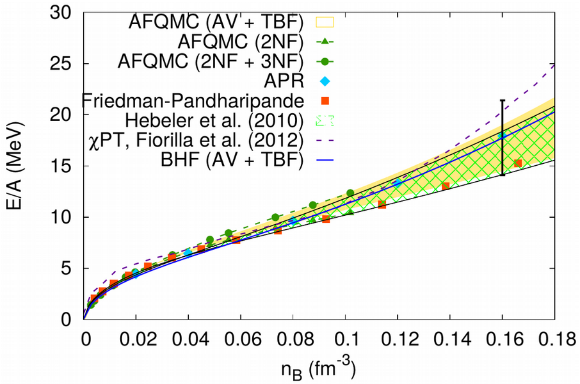


Equations are formally equivalent to the Time-Dependent HFB (TDHFB)

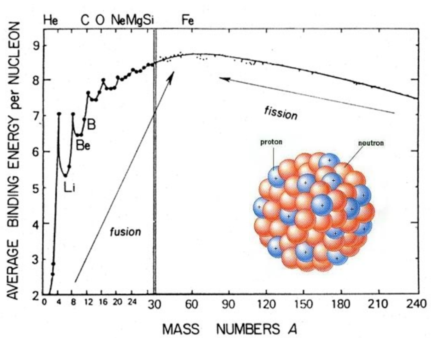
(non-interacting quasi-particles)

$$\langle\Psi|\hat{O}_{1\mathrm{B}}|\Psi\rangle = O_{1\mathrm{B}} = \langle\Phi|\hat{O}_{1\mathrm{B}}|\Phi\rangle$$

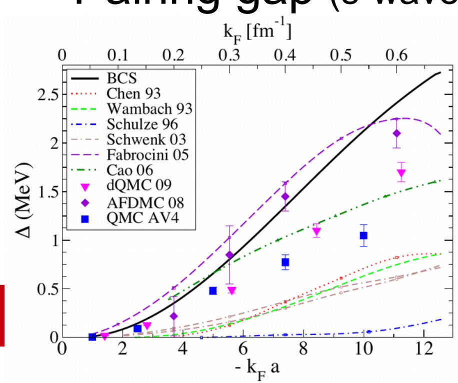
EoS (typically from QMC)



Exp. data for nuclei
(masses, radii, ...)



Pairing gap (s-wave)



Dimensional arguments, renormalizability, Galilean invariance, and symmetries (translational, rotational, gauge, parity)

Postulate

Input 1

Input 2

Input 3

Energy Density Functional E[n,...]

Validation against other quantities

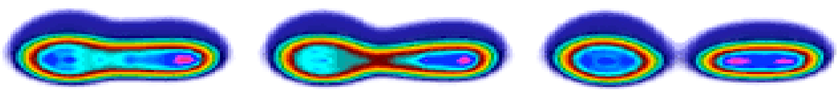
Predictions...

Quality of DFT results strongly depend on quality of the functional!

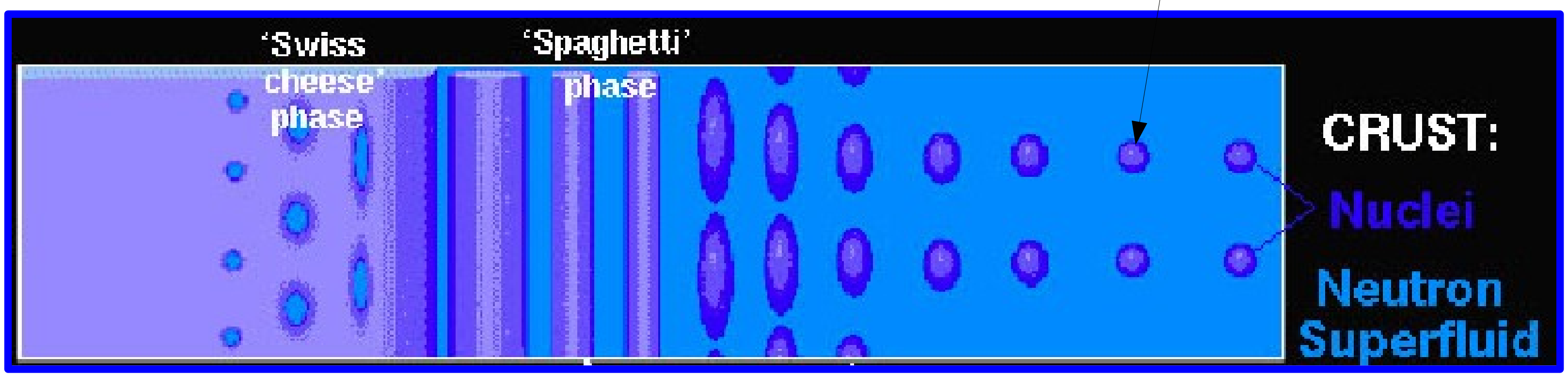
Accuracy of superfluid TDDFT was extensively tested over last years...

Example: fission of heavy nucleus

[Phys. Rev. Lett. 116, 122504 (2016)]



Nuclei - terrestrial experiments.



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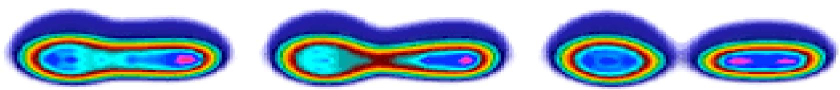
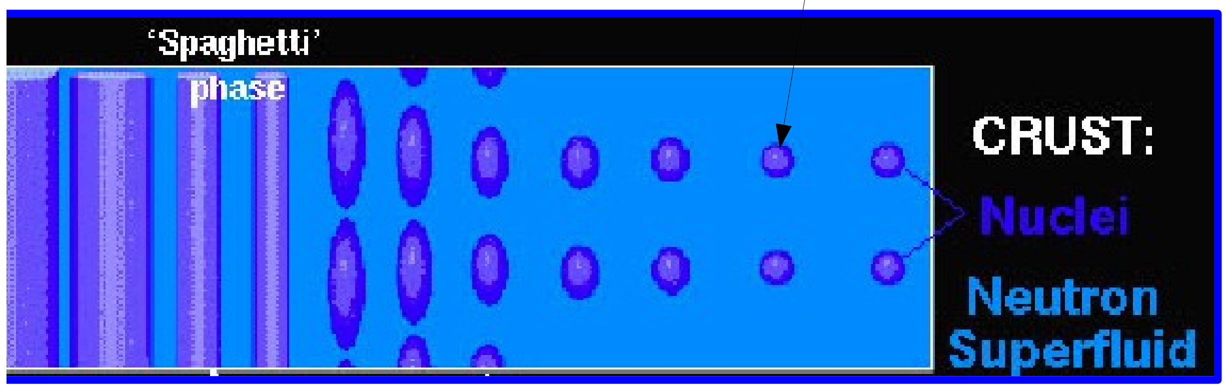


E^* (MeV)	E_n (MeV)	TKE_{TDSLDA} (MeV)	TKE_{syst} (MeV)	err (%)
8.08	1.542	173.81	177.26	1.95
9.60	3.063	174.73	176.73	1.13
10.10	3.560	179.09	176.56	1.43
10.57	4.032	173.67	176.39	1.55
10.58	4.043	173.39	176.39	1.70
10.58	4.047	175.11	176.39	0.72
10.60	4.065	174.75	176.38	0.92
11.07	4.534	176.46	176.22	0.14
11.56	5.024	175.15	176.05	0.51
12.05	5.515	176.75	175.88	0.49
12.15	5.610	176.36	175.84	0.29
12.16	5.626	176.10	175.84	0.15

Table 1: Total kinetic energy (TKE) as a functional of excitation energy E of ^{240}Pu obtained from TDSLDA compared (3rd column) with experimental data (4th column). The last column demonstrate present accuracy. Relative accuracy is at level 2% or better.*



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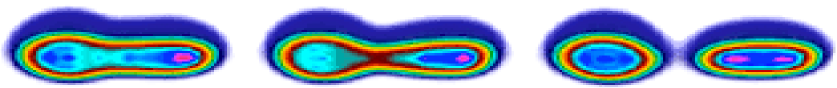

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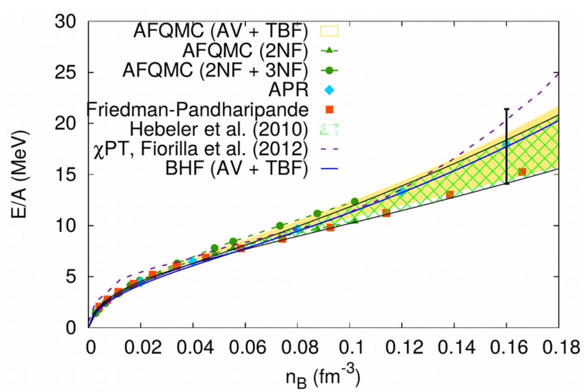
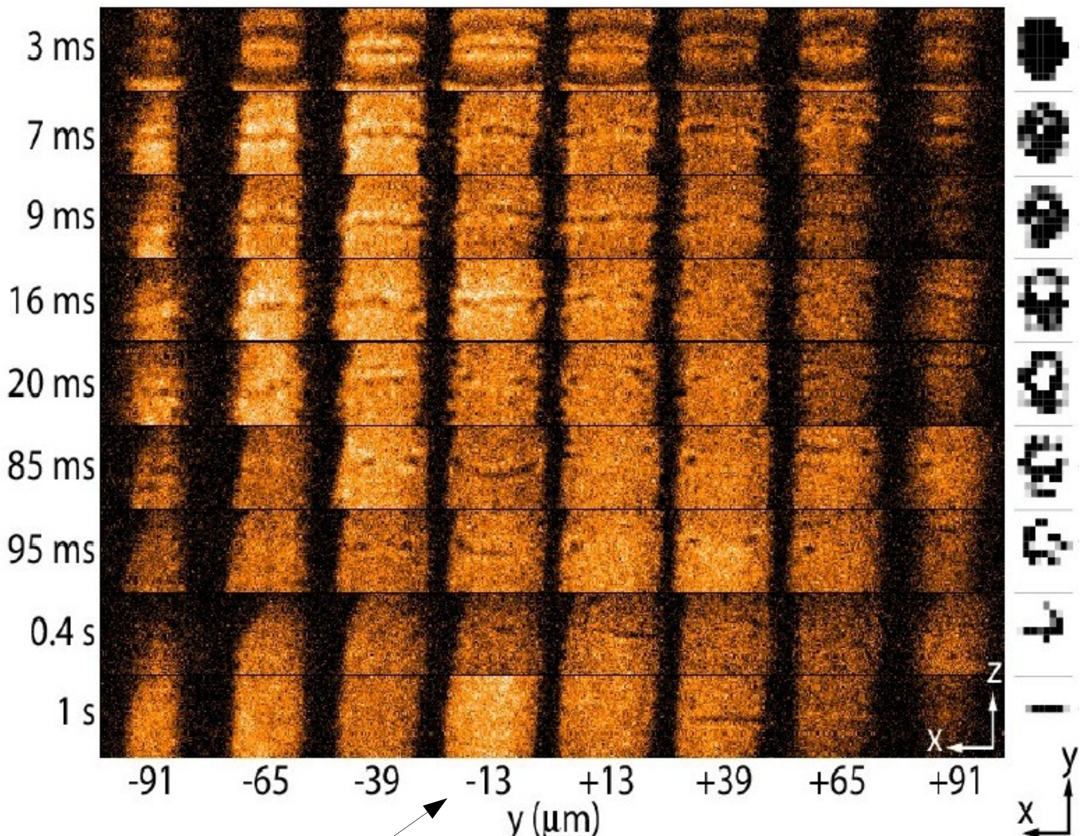


Dilute neutron matter very well constrained by QMC calculations.

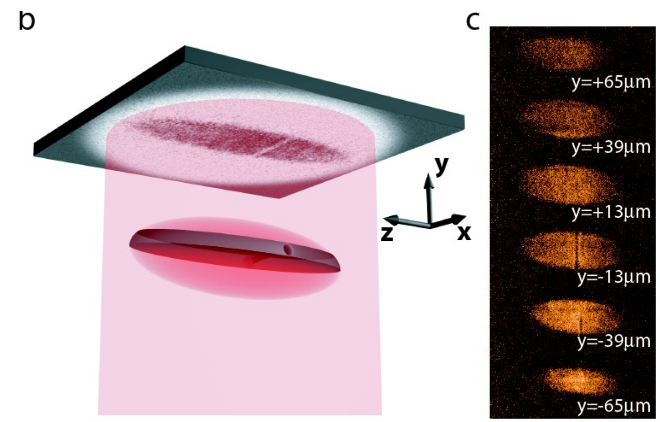
(a)

experiment

Phys. Rev. Lett. 116, 045304 (2016)

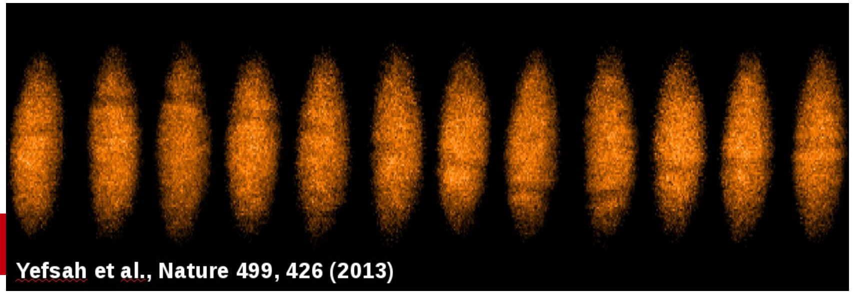


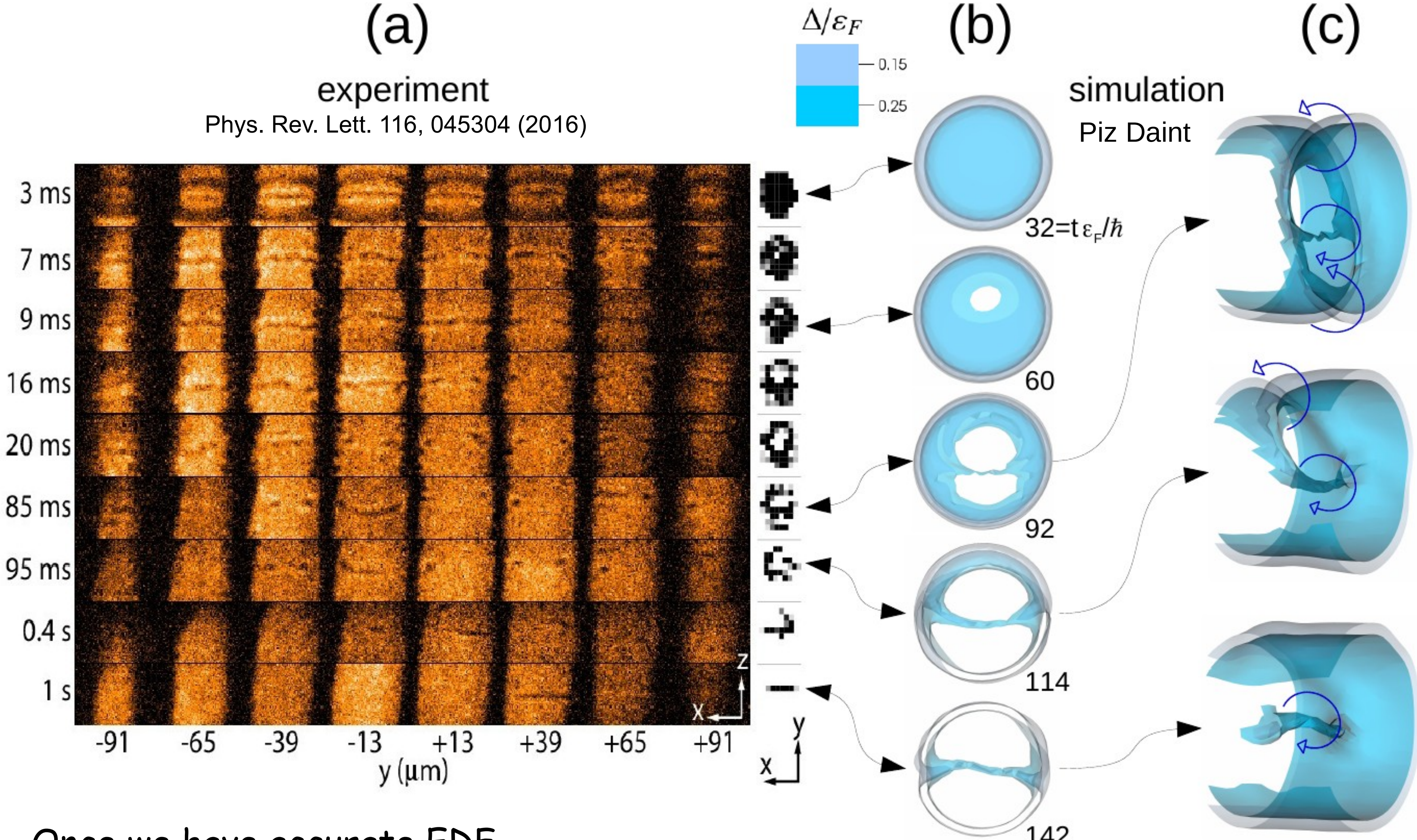
Series of MIT experiments:
Nature 499, 426 (2013);
PRL 113, 065301 (2014);
PRL 116, 045304 (2016);
→ observation of decay
of a dark soliton into a vortex line



unitary Fermi gas

(superfluid properties demonstrate here
in form of topological defects)

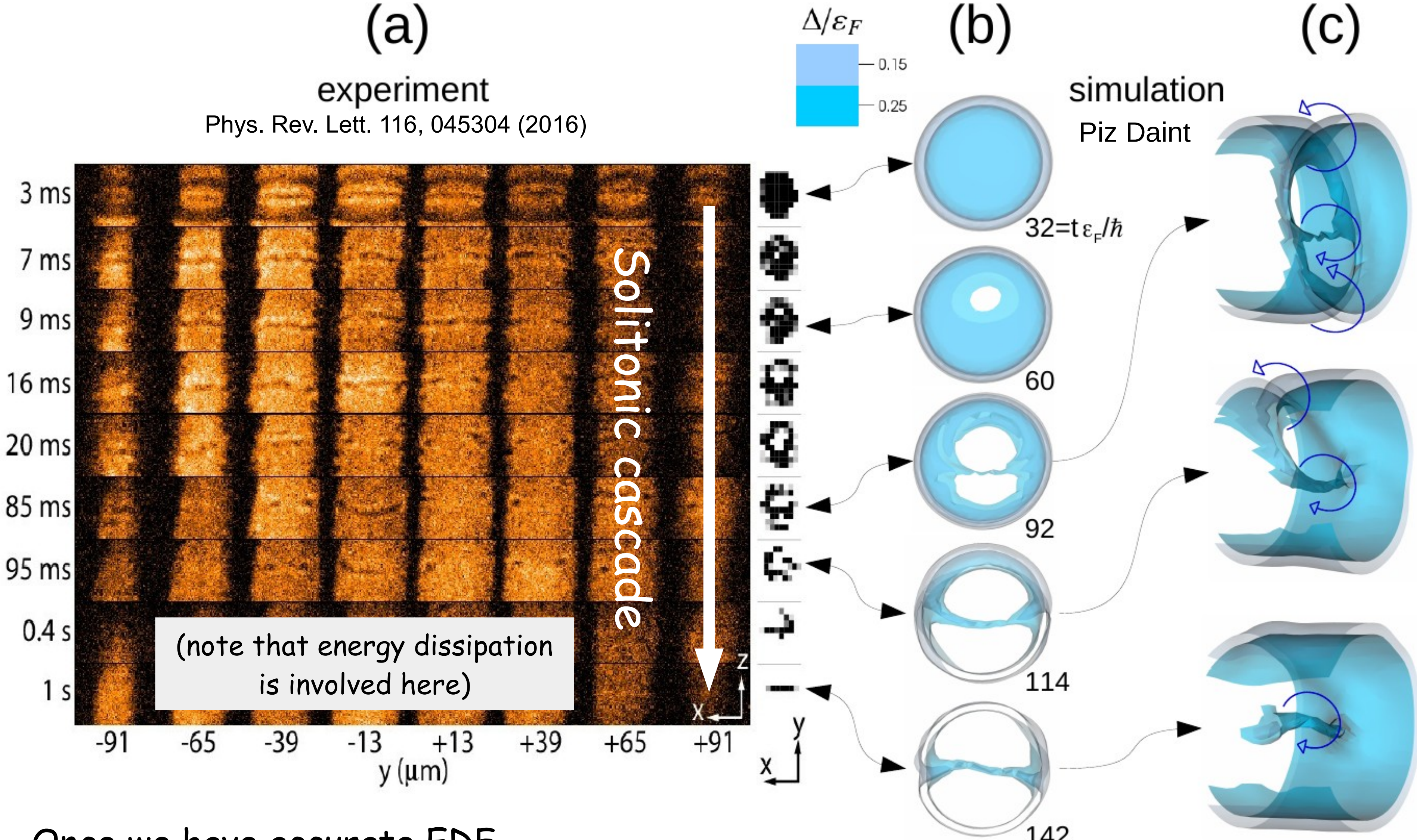




Once we have accurate EDF
→ remarkable agreement between theory and data!

G. Wlazłowski, K. Sekizawa, M. Marchwiany, P. Magierski, PRL 2018 (arXiv:1711.05803)

No adjusting parameters to the experiment!



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To execute superfluid TDDFT we need supercomputers...

TOP 500 The List.

Rank	System	Cores	Rmax (TFlop/s)	Rpeak (TFlop/s)	Power (kW)
1	**Sunway TaihuLight** - Sunway MPP, Sunway SW26010 260C 1.45GHz, Sunway , NRCPC National Supercomputing Center in Wuxi China	10,649,600	93,014.6	125,435.9	15,371
2	**Tianhe-2 (MilkyWay-2)** - TH-IVB-FEP Cluster, Intel Xeon E5-2692 12C 2.200GHz, TH Express-2, Intel Xeon Phi 31S1P , NUDT National Super Computer Center in Guangzhou China	3,120,000	33,862.7	54,902.4	17,808
3	**Piz Daint** - Cray XC50, Xeon E5-2690v3 12C 2.6GHz, Aries interconnect , NVIDIA Tesla P100 , Cray Inc. Swiss National Supercomputing Centre (CSCS) Switzerland	361,760	19,590.0	25,326.3	2,272 
4	**Gyoukou** - ZettaScaler-2.2 HPC system, Xeon D-1571 16C 1.3GHz, Infiniband EDR, PEZY-SC2 700Mhz , ExaScaler Japan Agency for Marine-Earth Science and Technology Japan	19,860,000	19,135.8	28,192.0	1,350
5	**Titan** - Cray XK7, Opteron 6274 16C 2.200GHz, Cray Gemini interconnect, NVIDIA K20x , Cray Inc. DOE/SC/Oak Ridge National Laboratory United States	560,640	17,590.0	27,112.5	8,209 
6	**Sequoia** - BlueGene/Q, Power BQC 16C 1.60 GHz, Custom , IBM DOE/NNSA/LLNL United States	1,572,864	17,173.2	20,132.7	7,890

To execute superfluid TDDFT we need supercomputers...

TOM SIMONITE BUSINESS 06.08.18 12:00 PM

THE US AGAIN HAS THE WORLD'S MOST POWERFUL SUPERCOMPUTER

The IBM-built Summit supercomputer is the world's smartest and most powerful AI machine. Its racks are connected by over 185 miles of fiber-optic cables.

GENEVIEVE MARTIN/OAK RIDGE NATIONAL LABORATORY

Rank	System	Cores	Rmax (TFlop/s)	Rpeak (TFlop/s)	Power (kW)
		00	93,014.6	125,435.9	15,371
		00	33,862.7	54,902.4	17,808
		60	19,590.0	25,326.3	2,272
		00	19,135.8	28,192.0	1,350
		40	17,590.0	27,112.5	8,209
6	**Sequoia** - BlueGene/Q, Power BQC 16C 1.60 GHz, Custom , IBM DOE/NNSA/LLNL United States	1,572,864	17,173.2	20,132.7	7,890

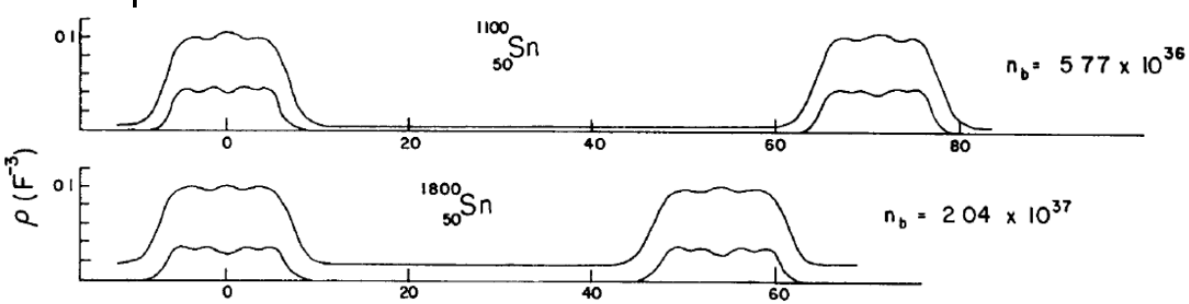
To execute superfluid TDDFT we need supercomputers...

Present computing capabilities:

(for nuclear systems)

- full 3D (unconstrained) dynamics
- volumes up to 100^3 fm^3
- number of particles of order 10^4
- up to 10^6 time steps (for nuclear systems it gives trajectory length 10^{-19} sec)

Rank	System	Cores	Rmax (TFlop/s)	Rpeak (TFlop/s)	Power (kW)
1	Sunway TaihuLight - Sunway 1.45GHz, Sunway , NRCPC National Supercomputing Ce China			125,435.9	15,371
2	Tianhe-2 (MilkyWay-2) - TH- 12C 2.200GHz, TH Express-2 National Super Computer Ce China			54,902.4	17,808
3	Piz Daint - Cray XC50, Xeon E interconnect , NVIDIA Tesla P Swiss National Supercomput Switzerland			25,326.3	2,272
4	Gyoukou - ZettaScaler-2.2 H Infiniband EDR, PEZY-SC2 70 Japan Agency for Marine-Ear Japan			28,192.0	1,350
		,640	17,590.0	27,112.5	8,209
		,864	17,173.2	20,132.7	7,890



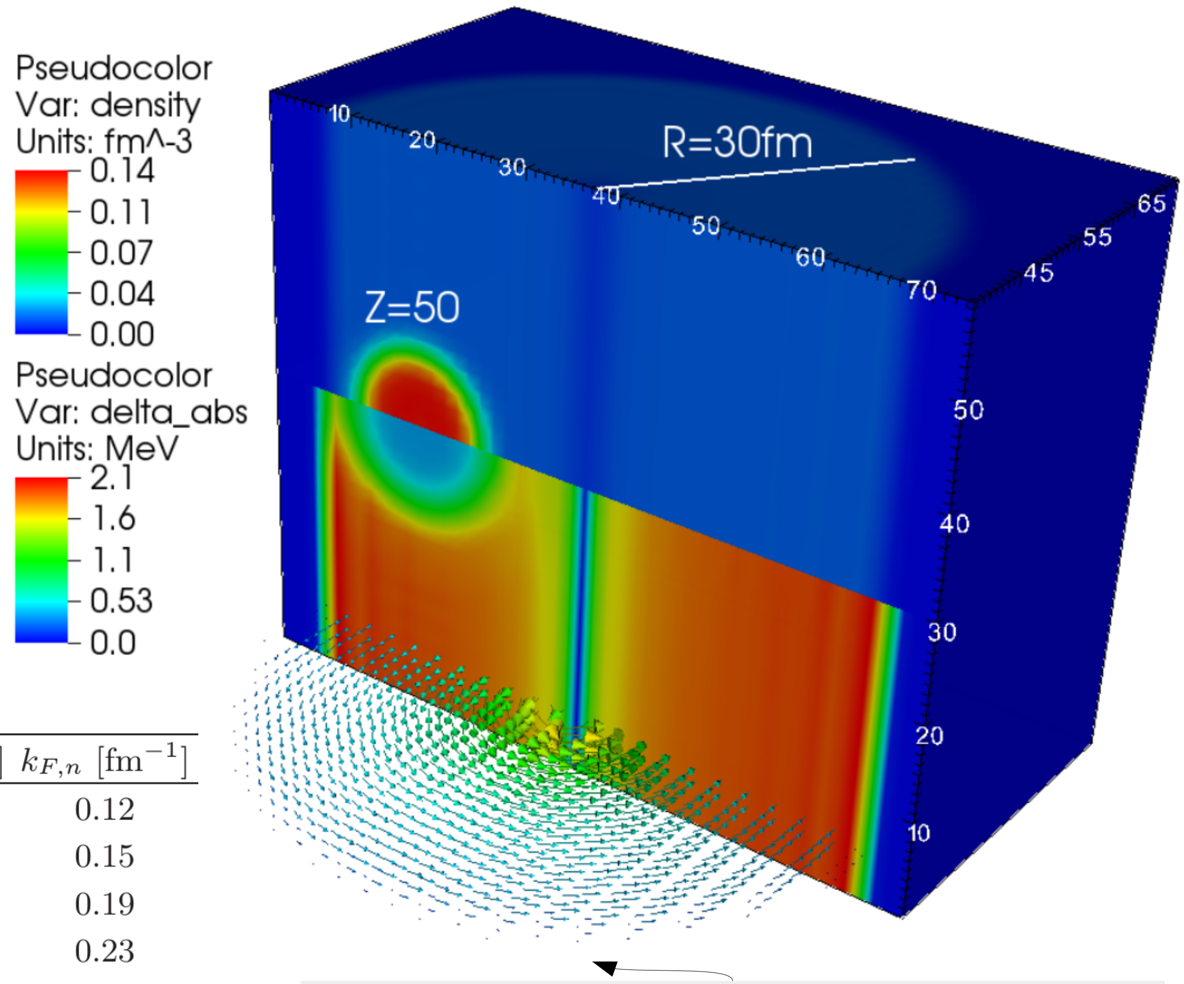
Negele&Vautherin,Nucl.Phys.A207,298(1973)

Example
of initial configuration...

G. Wlazłowski, K. Sekizawa, P. Magierski, A. Bulgac, M.M. Forbes,
Phys. Rev. Lett. 117, 232701 (2016)

Self-consistent solution of static problem with constraints:

- fixed center of mass of protons
- nonzero total angular momentum of neutrons



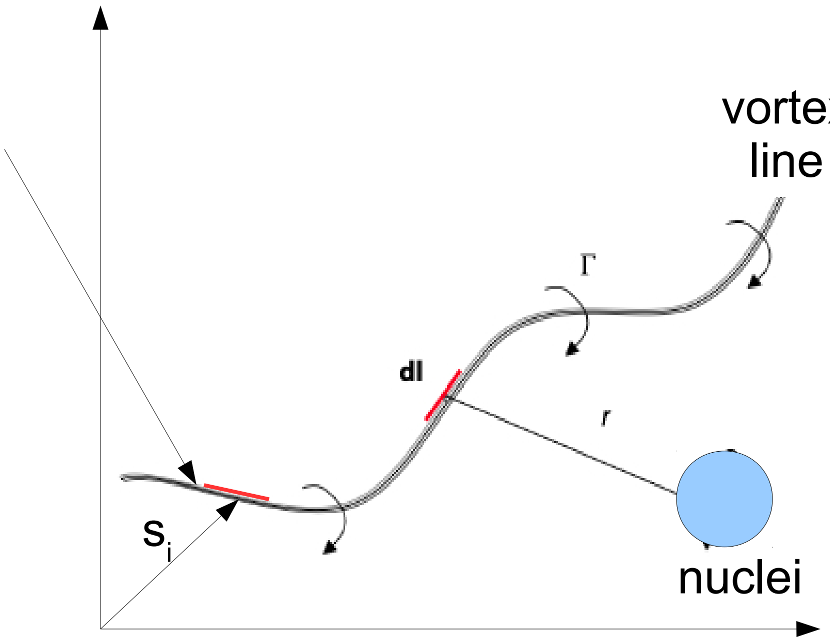
J. Negele and D. Vautherin, Nucl. Phys. A 207, 298 (1973)

Zone	Element	Z	N	R_{WS} [fm]	ρ_b [g · cm^{-3}]	$k_{F,n}$ [fm^{-1}]
11	^{180}Zr	40	140	53.6	$4.67 \cdot 10^{11}$	0.12
10	^{200}Zr	40	160	49.2	$6.69 \cdot 10^{11}$	0.15
9	^{250}Zr	40	210	46.4	$1.00 \cdot 10^{12}$	0.19
8	^{320}Zr	40	280	44.4	$1.47 \cdot 10^{12}$	0.23
7	^{500}Zr	40	460	42.2	$2.66 \cdot 10^{12}$	0.31
6	^{950}Sn	50	900	39.3	$6.24 \cdot 10^{12}$	0.43
5	^{1100}Sn	50	1050	35.7	$9.65 \cdot 10^{12}$	0.51
4	^{1350}Sn	50	1300	33.0	$1.49 \cdot 10^{13}$	0.60
3	^{1800}Sn	50	1750	27.6	$3.41 \cdot 10^{13}$	0.80
2	^{1500}Zr	40	1460	19.6	$7.94 \cdot 10^{13}$	1.08
1	^{982}Ge	32	950	14.4	$1.32 \cdot 10^{14}$	1.33

Lowest energy state (constrained) for Z=50 and N=2,530 confined in tube of radius R=30fm

Vortex Filament Model

- *Each filament of the vortex line generates rotational flow around it,*
- *The total flow at arbitrary position can be calculated by means of Biot-Savart,*



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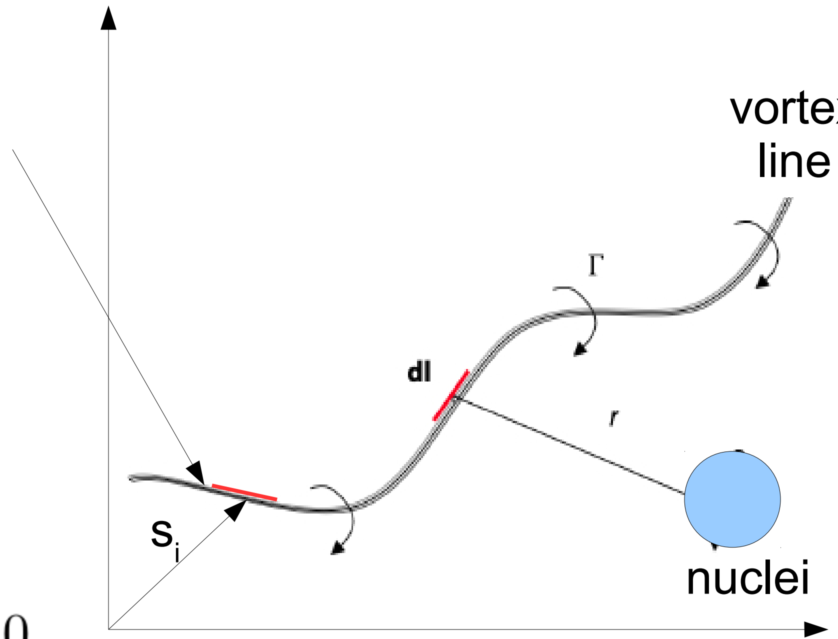
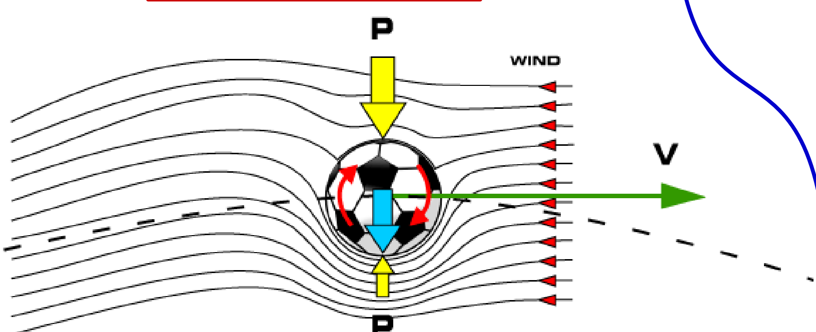


<u>Equation of motion for the vortex line:</u>

Balance of forces (mass of vortex negligible):

$$\kappa\rho_s\hat{\boldsymbol{t}}\times(\dot{\boldsymbol{s}}-\boldsymbol{v}_{ind}-\boldsymbol{v}_{ext})+\boldsymbol{f}^{VN}+\boldsymbol{f}^{D}=0$$

Magnus force



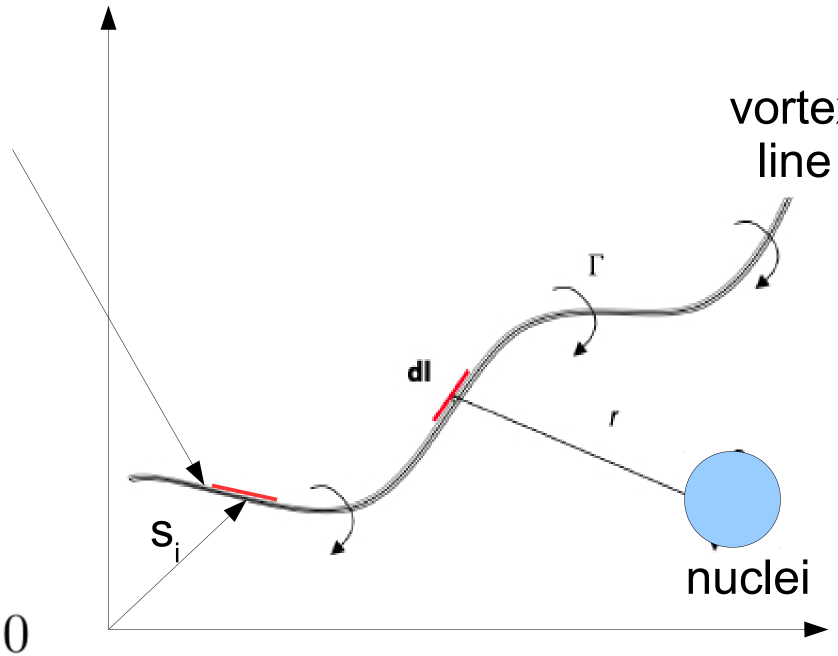
Vortex – nucleus interaction

“Dissipation” force

Example: $\boldsymbol{f}^{D}=-\eta\dot{\boldsymbol{s}}$

Vortex Filament Model (VFM)

- *Each filament of the vortex line generates rotational flow around it,*
- *The total flow at arbitrary position can be calculated by means of Biot-Savart,*



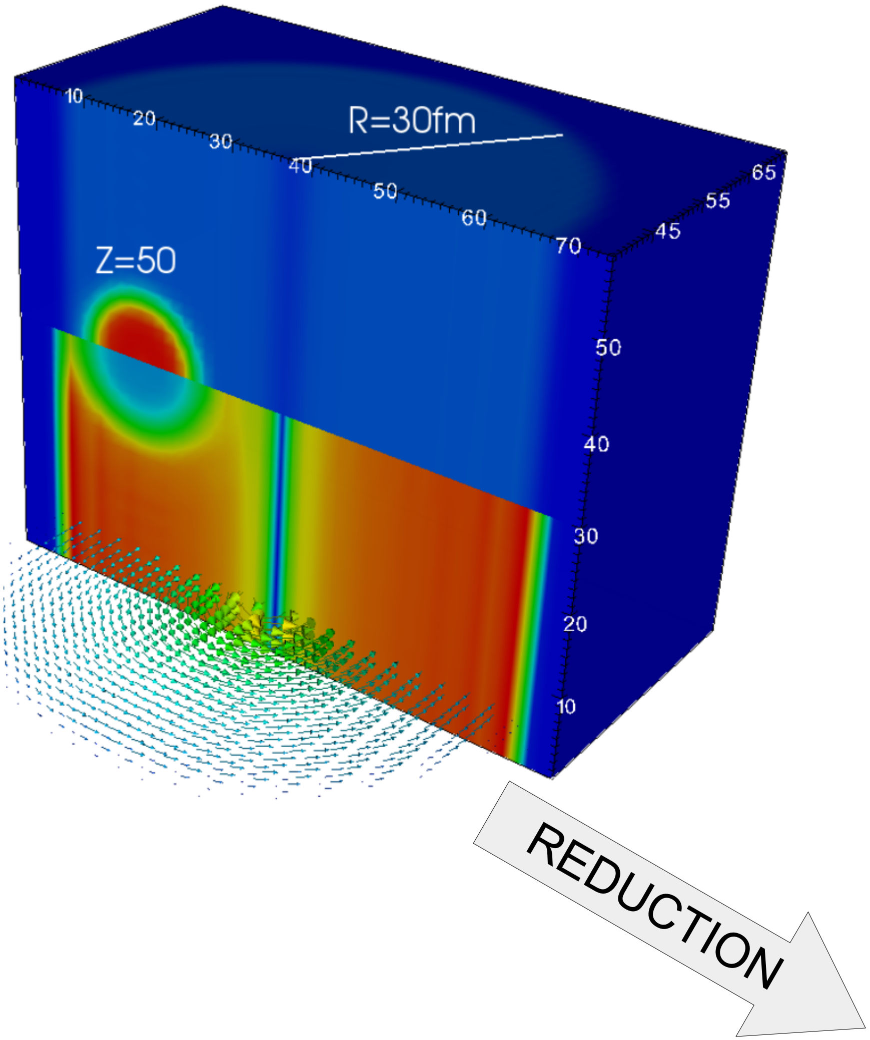
<u>Equation of motion for the vortex line:</u>

Balance of forces (mass of vortex negligible):

$$\kappa \rho_s \hat{\boldsymbol{t}} \times (\dot{\boldsymbol{s}} - \boldsymbol{v}_{ind} - \boldsymbol{v}_{ext}) + \boldsymbol{f}^{VN} + \boldsymbol{f}^{D} = 0$$

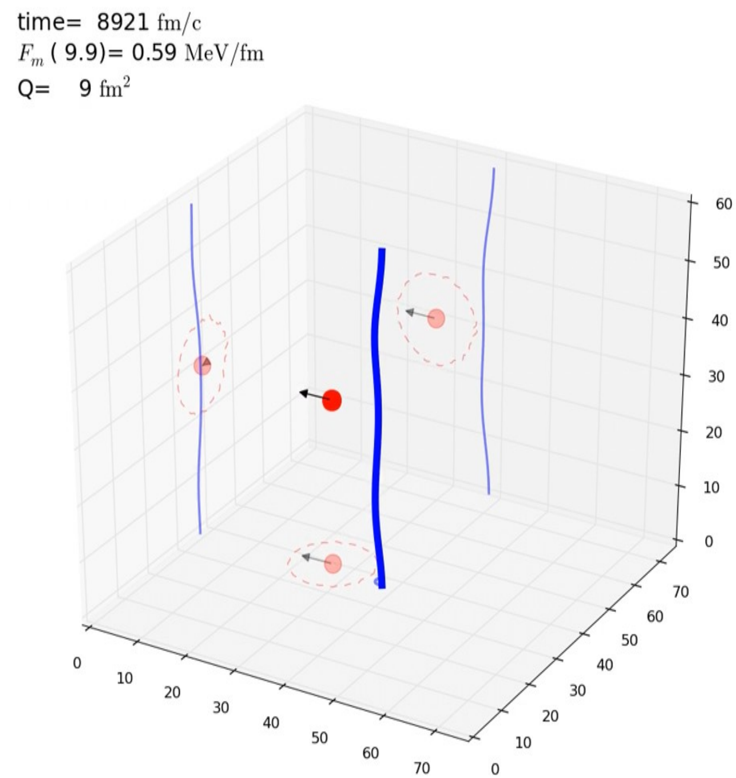
Our aim:

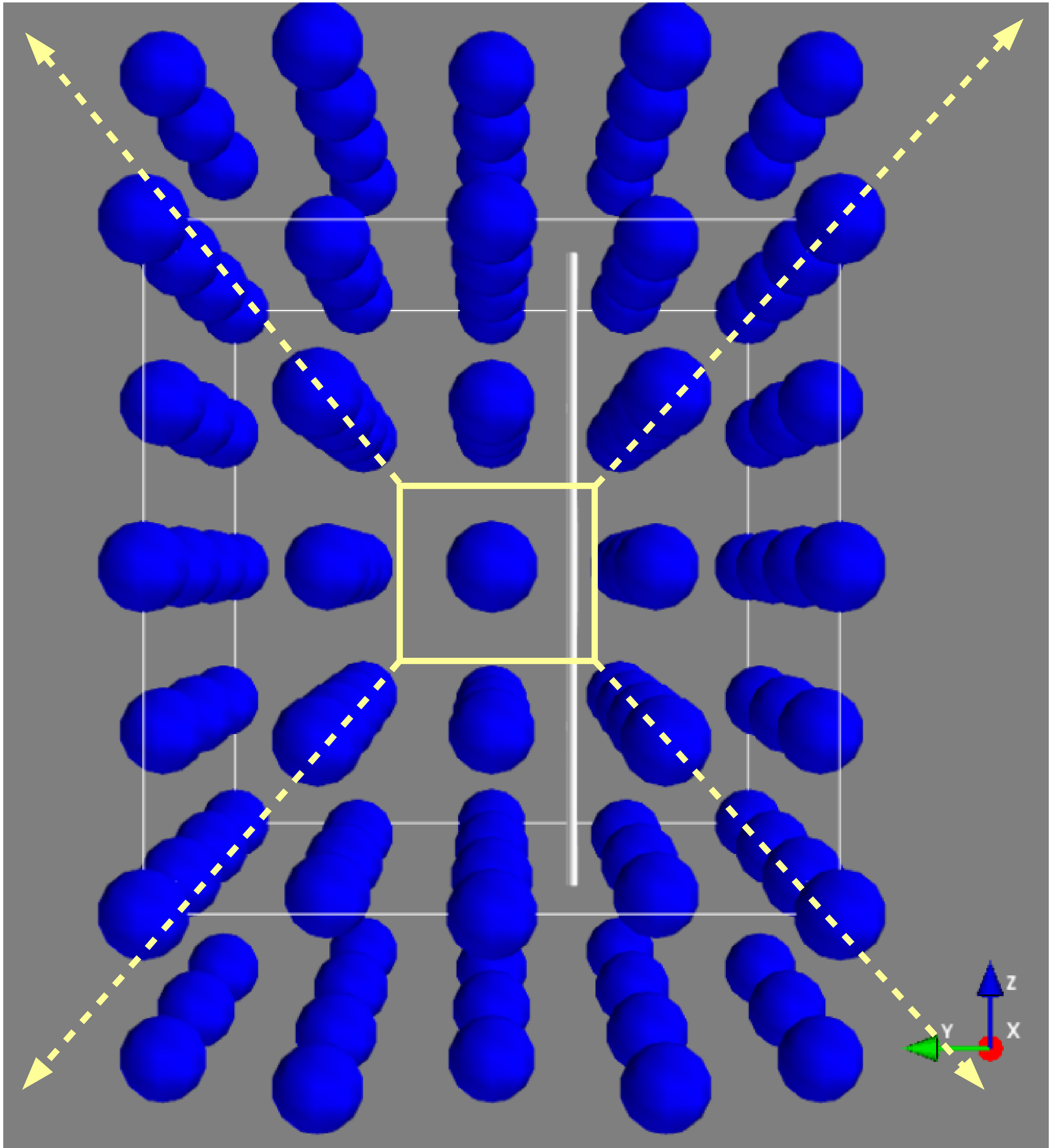
Construct such VFM that reproduces dynamics seen in microscopic simulations



Vortex Filament Model should reproduce it

(at qualitative and quantitative level)

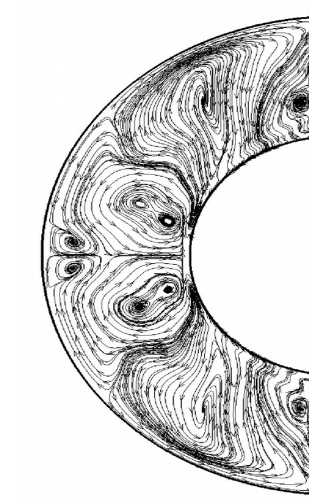




"Fluid element"

feed

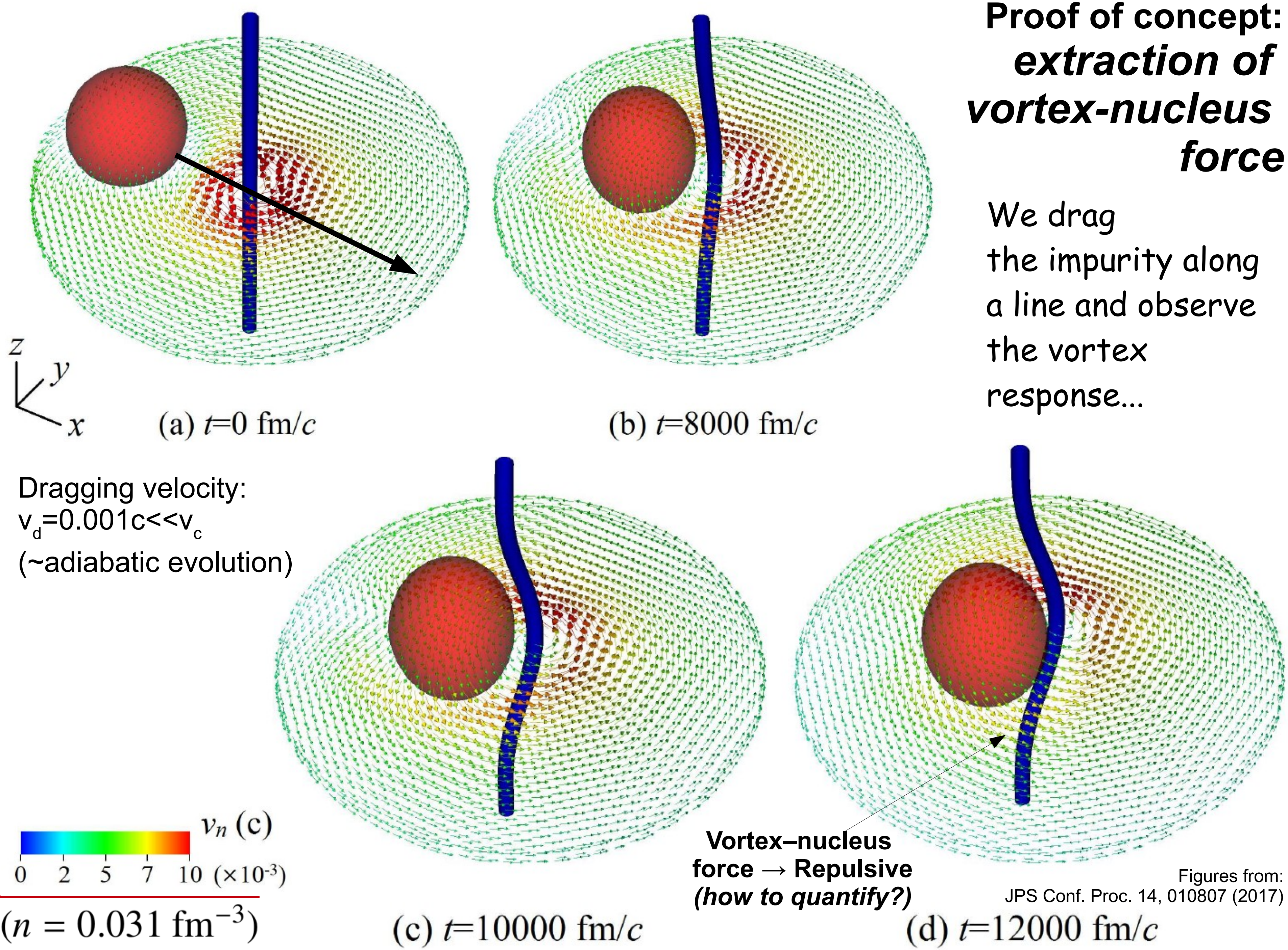
Two-fluid hydrodynamics



Proof of concept: *extraction of vortex-nucleus force*

We drag the impurity along a line and observe the vortex response...

Dragging velocity:
$v_d = 0.001c \ll v_c$
(~adiabatic evolution)



$(n = 0.031\ \mathrm{fm}^{-3})$

Vortex–nucleus force → Repulsive
(how to quantify?)

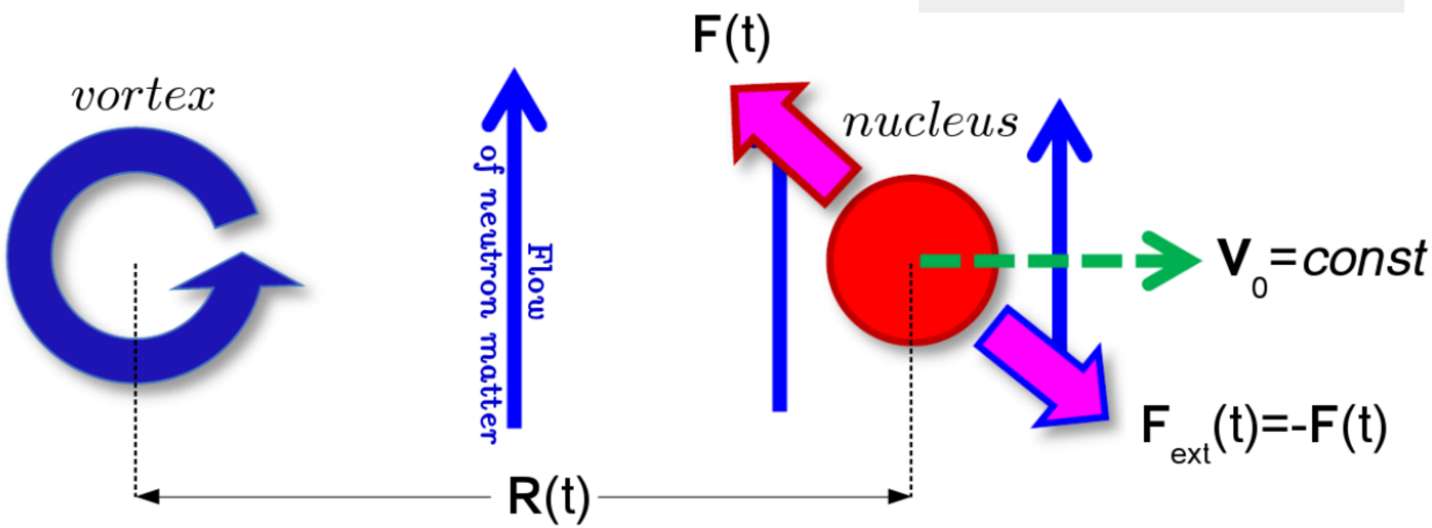
Figures from:
JPS Conf. Proc. 14, 010807 (2017)

We use Newton's 3rd law and extract the force from motion of the nucleus.....
Consider Eq. of motion for impurity:

$$M\frac{d\boldsymbol{v}}{dt} = \boldsymbol{F}_{\text{tot.}} = \boldsymbol{F} + \boldsymbol{F}_{\text{ext.}} = 0 \Leftrightarrow \boldsymbol{v} = \text{const}$$

Vortex-nucleus interaction

Uniform electric field that we control



We use Newton's 3rd law and extract the force from motion of the nucleus.....
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Vortex-nucleus interaction

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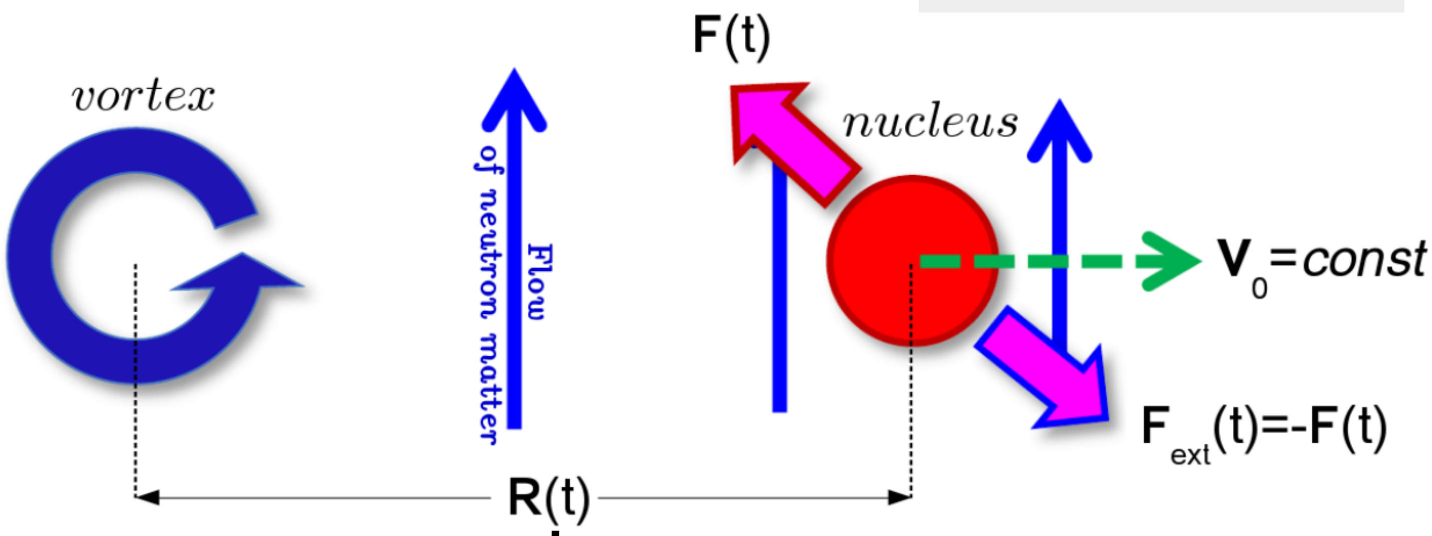


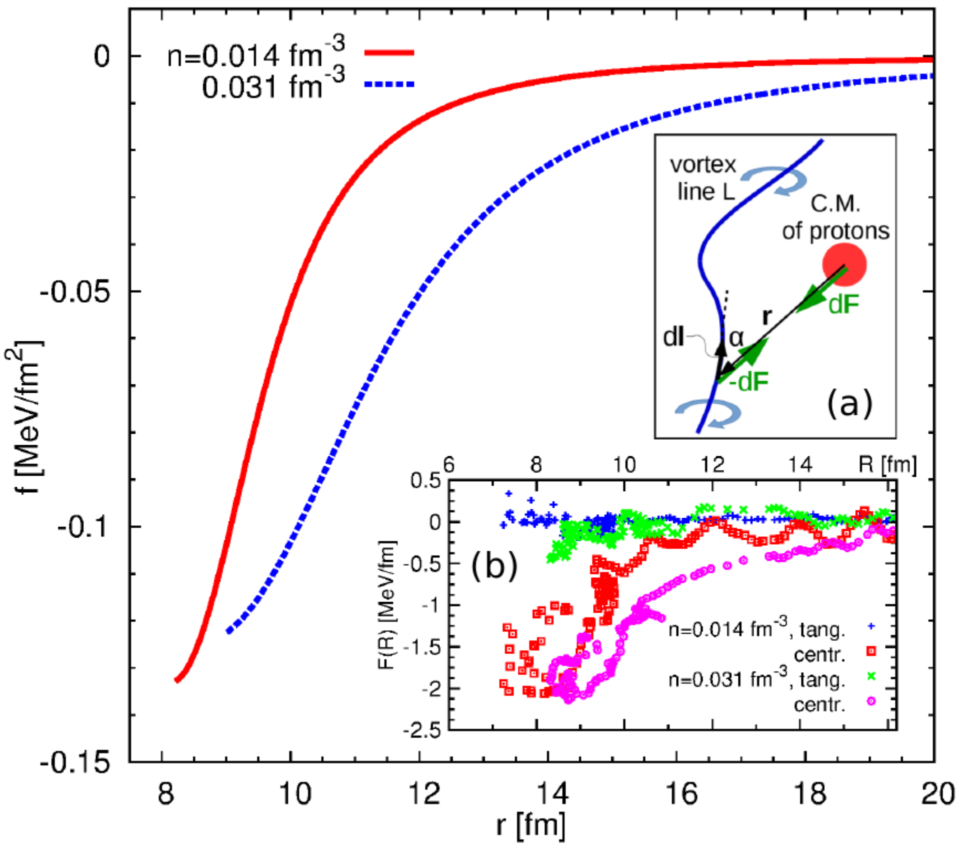
$$\boldsymbol{F}(R)$$

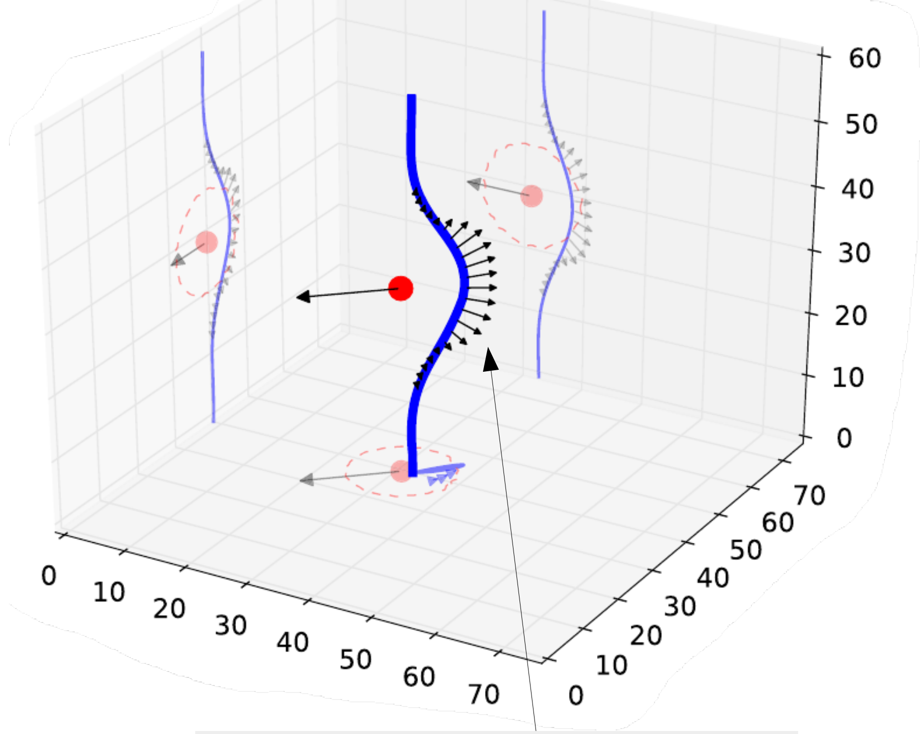
We construct F_{ext} dynamically:

$$\boldsymbol{F}_{\text{ext}}(t+\Delta t) = \boldsymbol{F}_{\text{ext}}(t) - \alpha\left[\boldsymbol{v}(t) - \boldsymbol{v}_0\right]$$

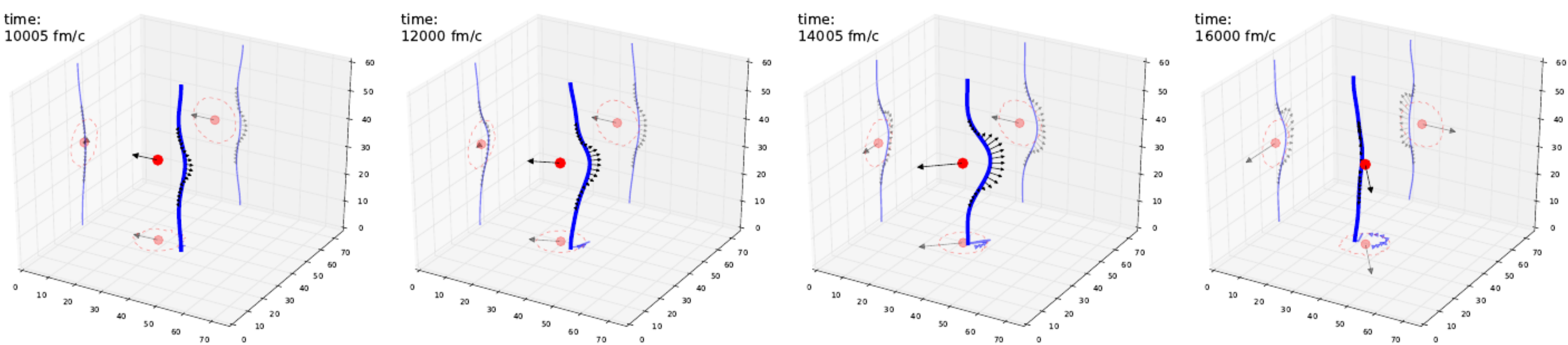
Phys. Rev. Lett. 117, 232701 (2016)

Force per unit length



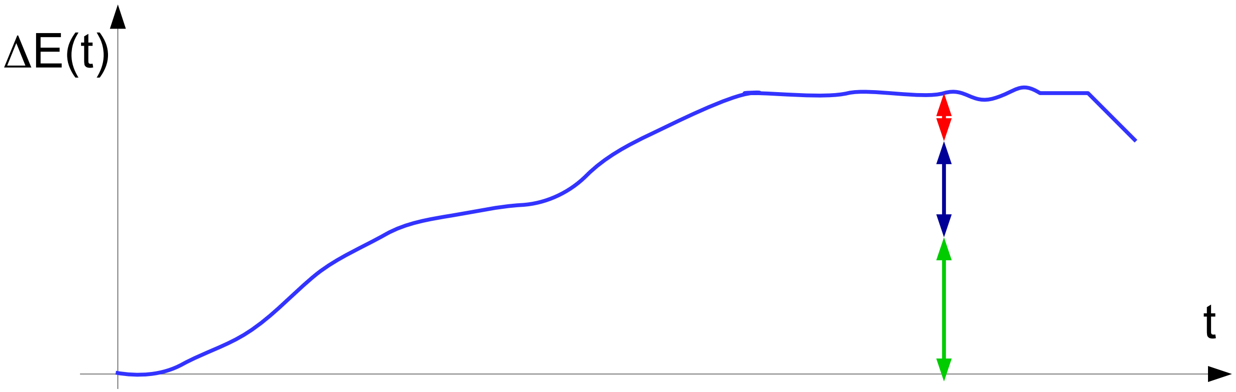


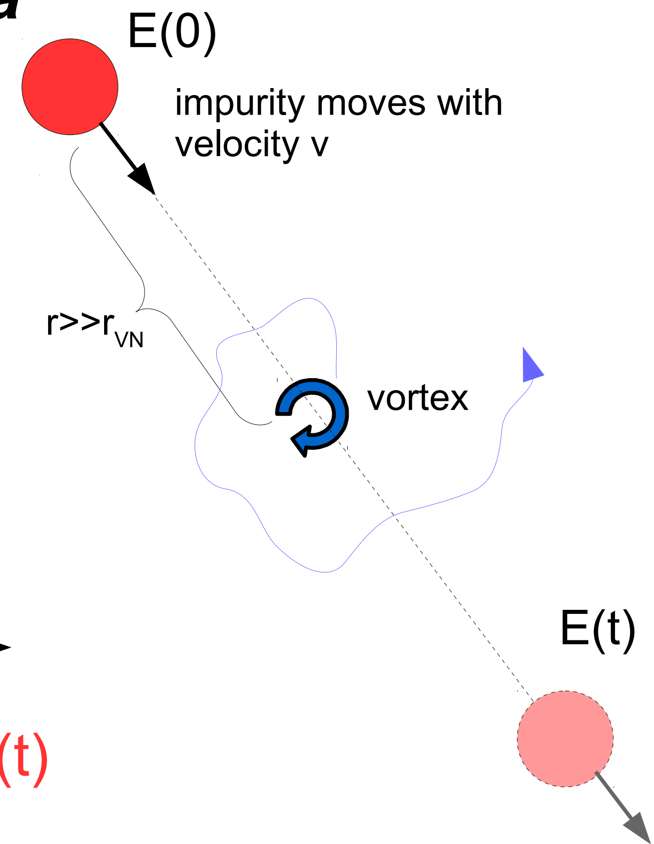
Only part close to nucleus surface contributes...



Extraction of the dissipation force – idea

- drag impurity with various velocities...
- look at excitation energy...

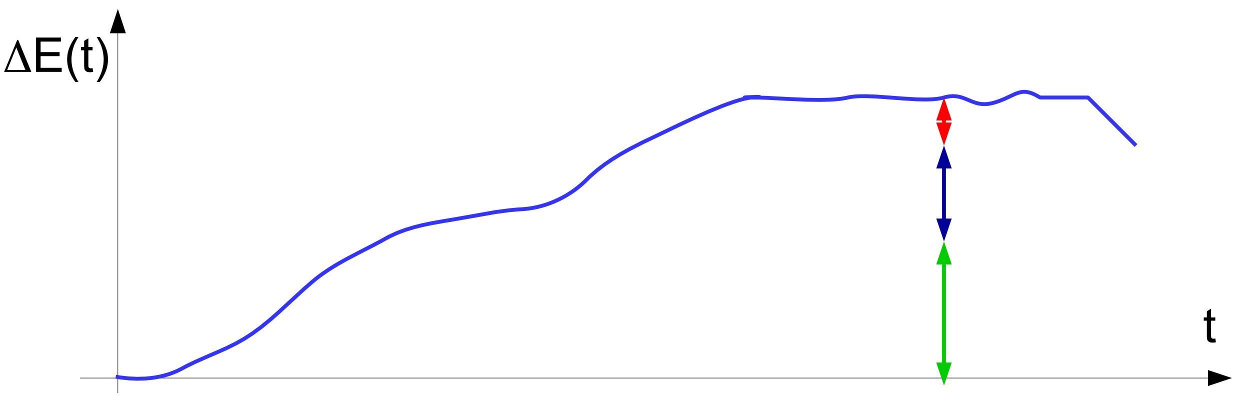


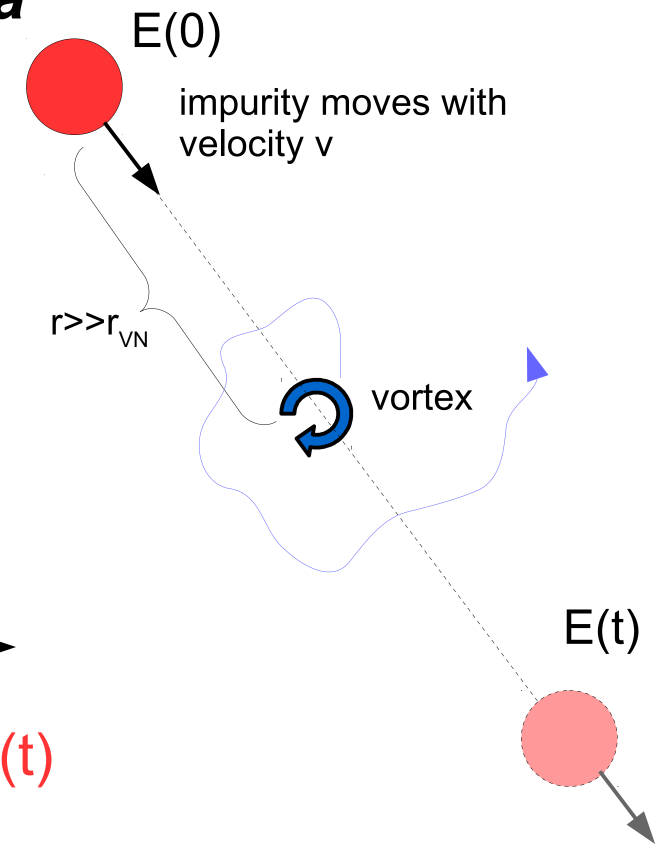


$$\Delta E(t) = \Delta E_{vortex}(t) + \Delta E_{compressible}(t) + \Delta E_{internal}(t)$$

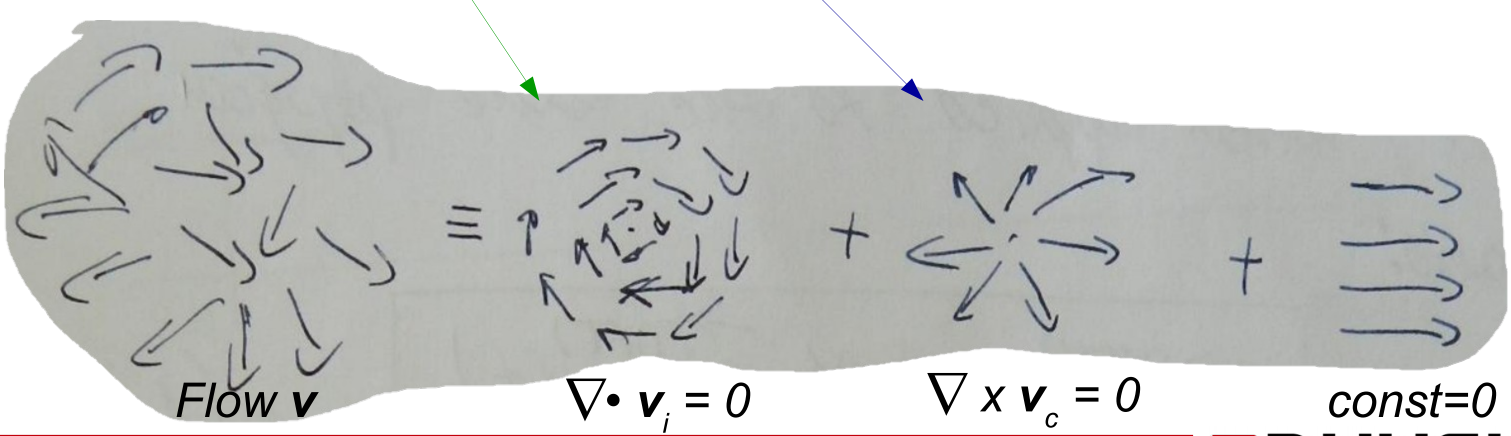
Extraction of the dissipation force – idea

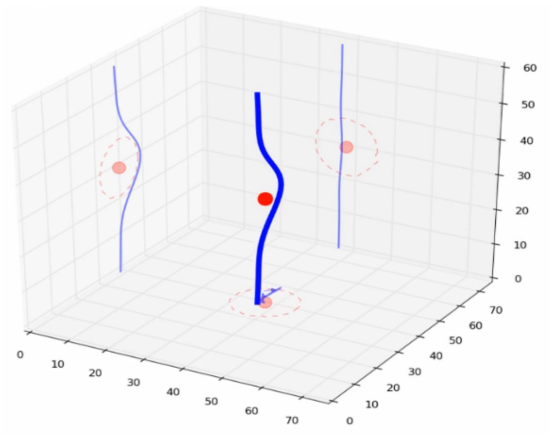
- drag impurity with various velocities...
- look at excitation energy...





$$\Delta E(t) = \Delta E_{vortex}(t) + \Delta E_{compressible}(t) + \Delta E_{internal}(t)$$





$$f^D = -\eta_1(\dot{s} - v_{\mathrm{nuclei}}) - \eta_2(\dot{s} - v_{\mathrm{ext}})$$

Use “ansatz” here for the dissipation force, and try to reproduce $W_d(t)$...

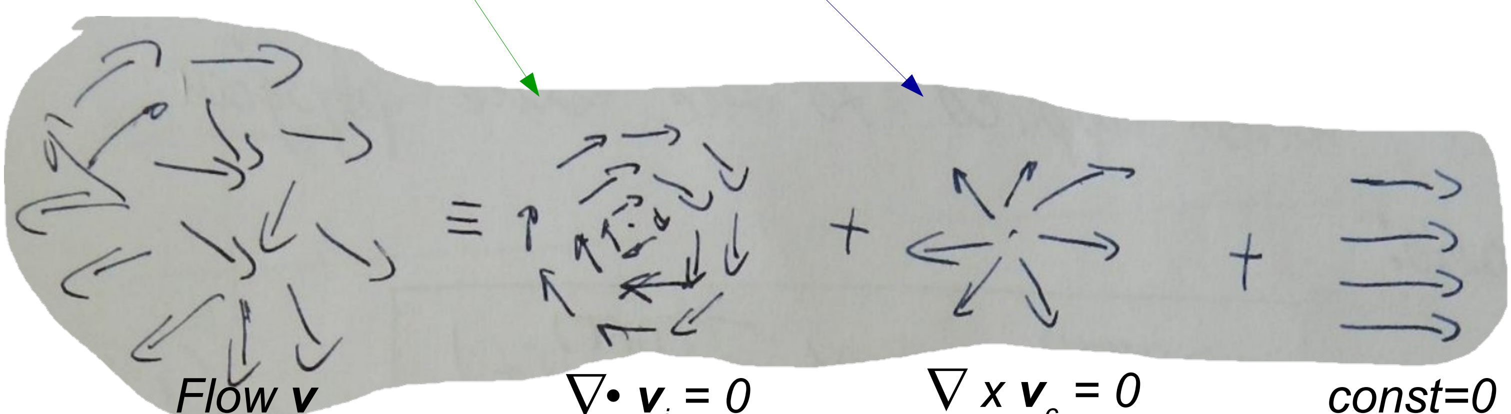
$$W_d(t) = \int_{\mathcal{L}} \left[\int_0^t f^D(t') \cdot \dot{s}_l(t')\, dt' \right] dl$$

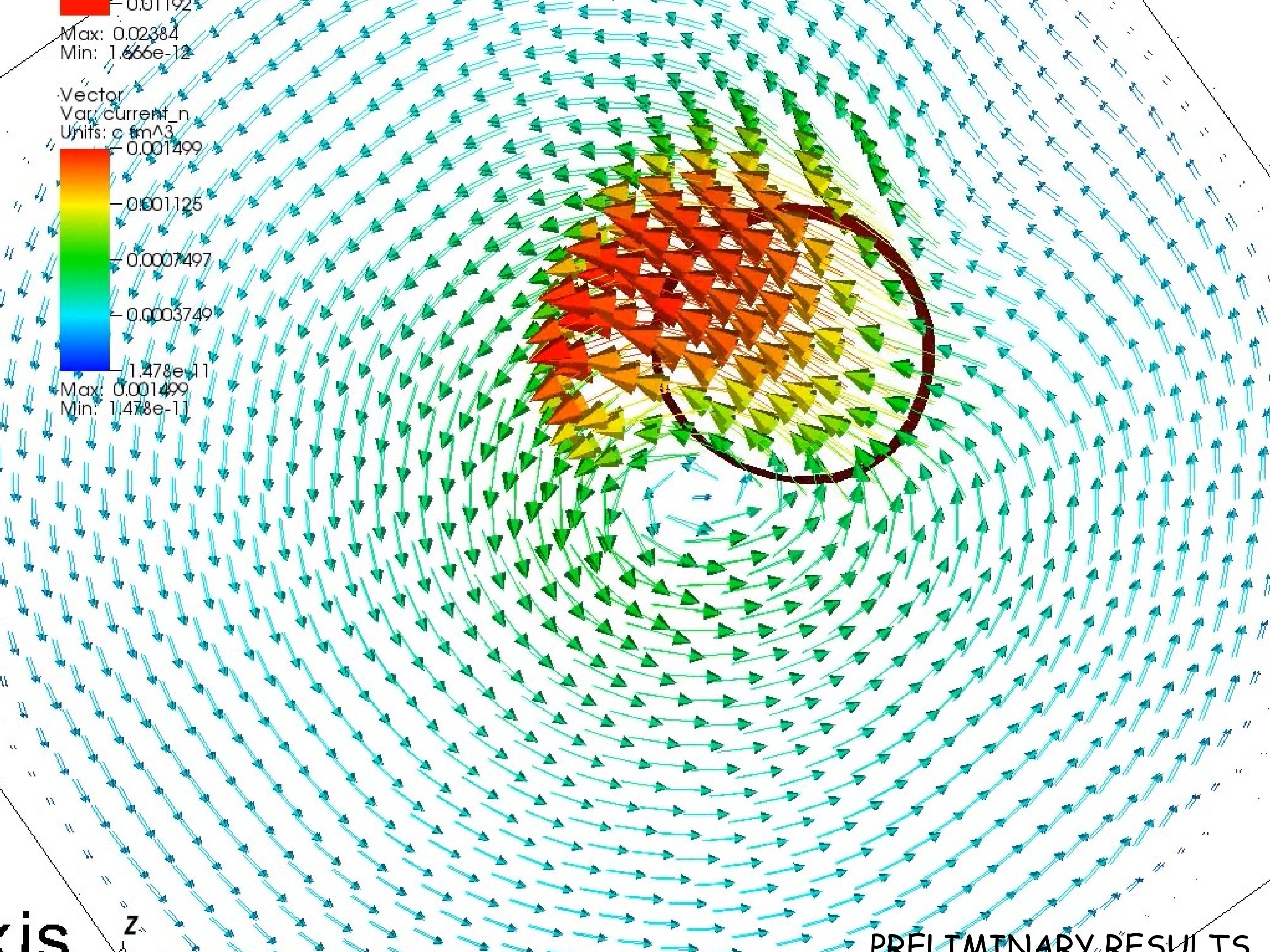
Reflected by the vortex line deformation in VFM

$W_d(t)$ = Dissipated energy (from point of view of VFM)

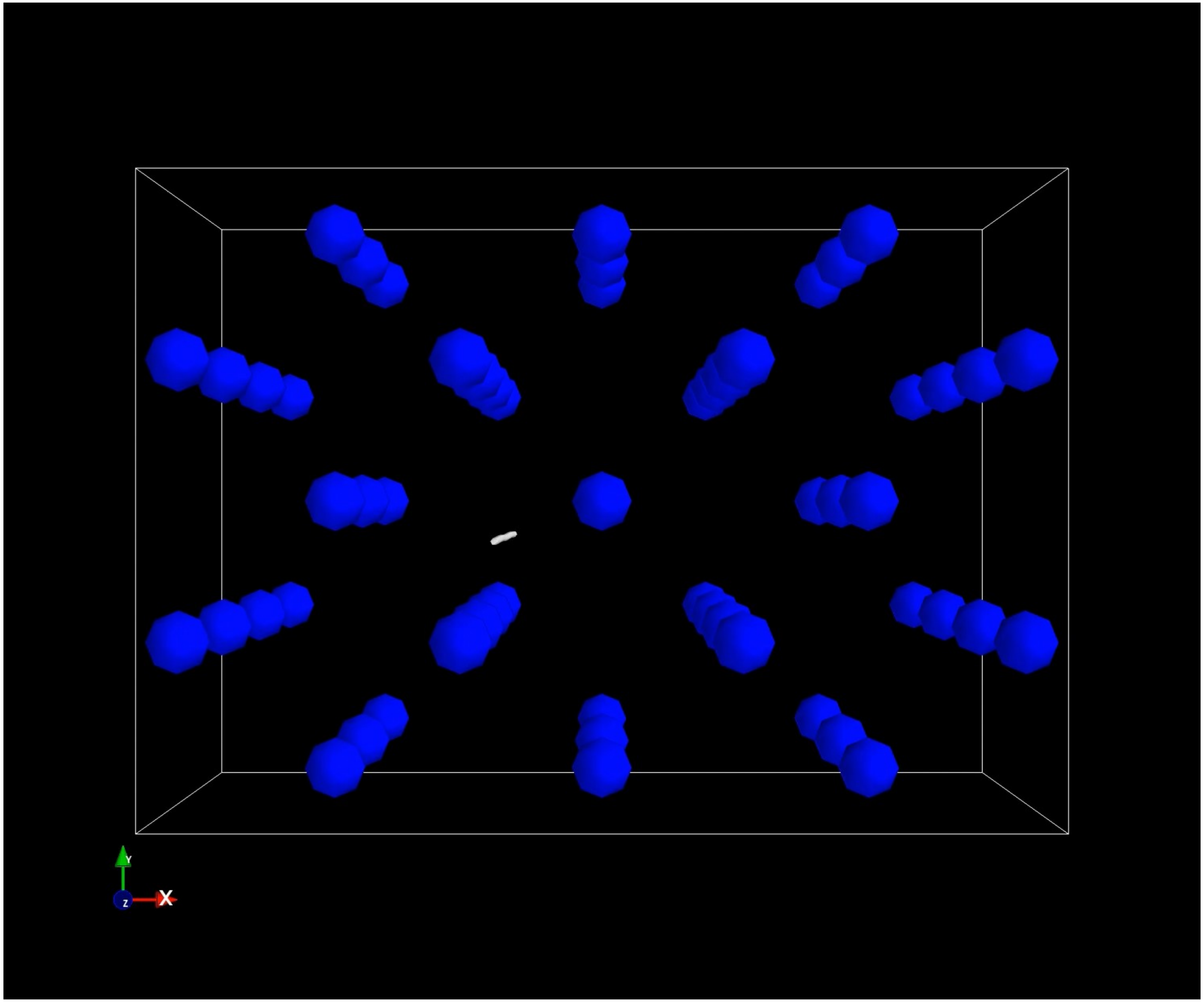
Known from simulations...

$$\Delta E(t) = \Delta E_{\mathrm{vortex}}(t) + \Delta E_{\mathrm{compressible}}(t) + \Delta E_{\mathrm{internal}}(t)$$



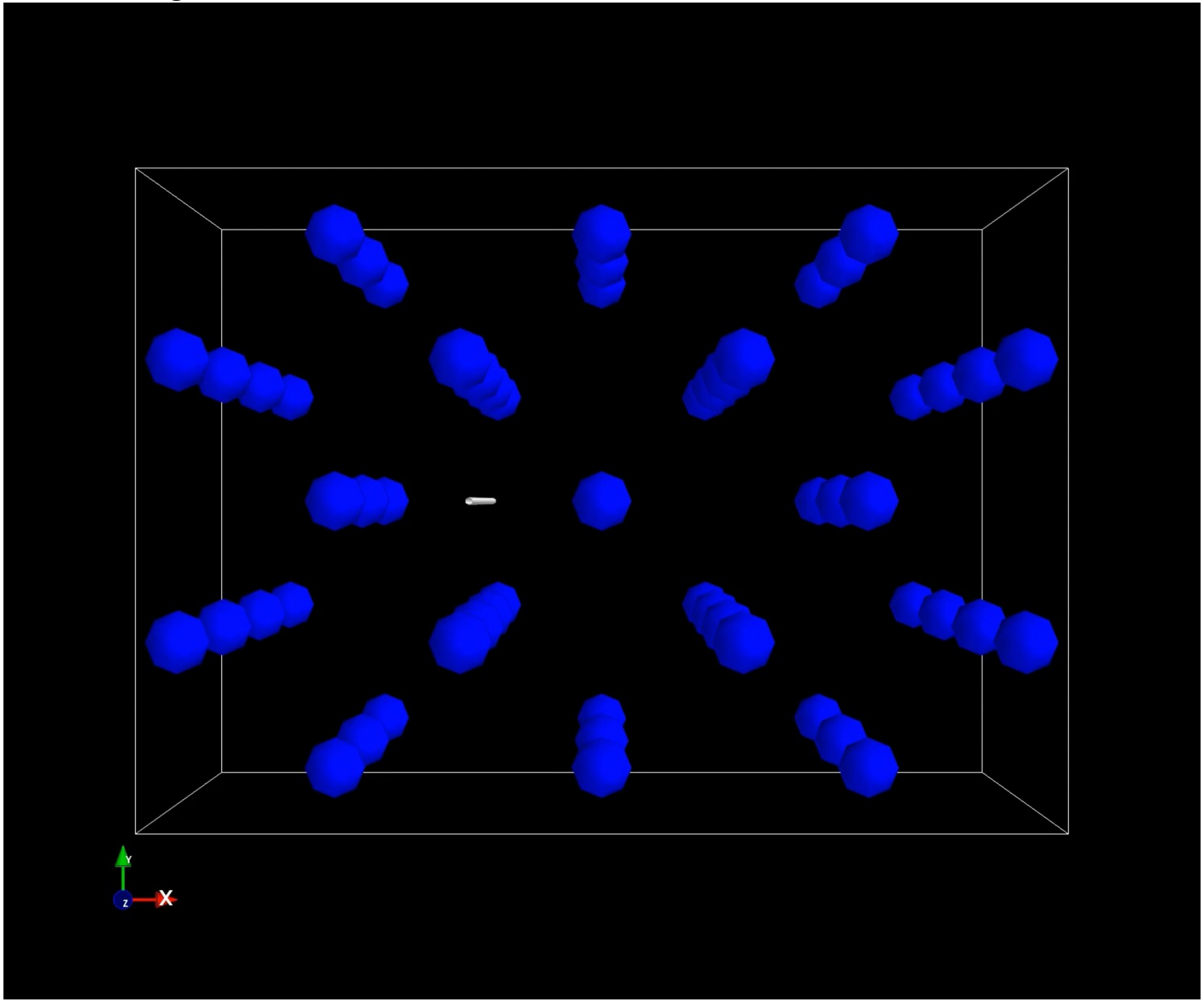

Max: 0.02384
Min: 1.666e-12
Vector
Var: current_n
Units: c fm^3
0.001499
0.001125
0.0007497
0.0003749
1.478e-11
Max: 0.001499
Min: 1.478e-11
Z
PRELIMINARY RESULTS

Put all elements together...



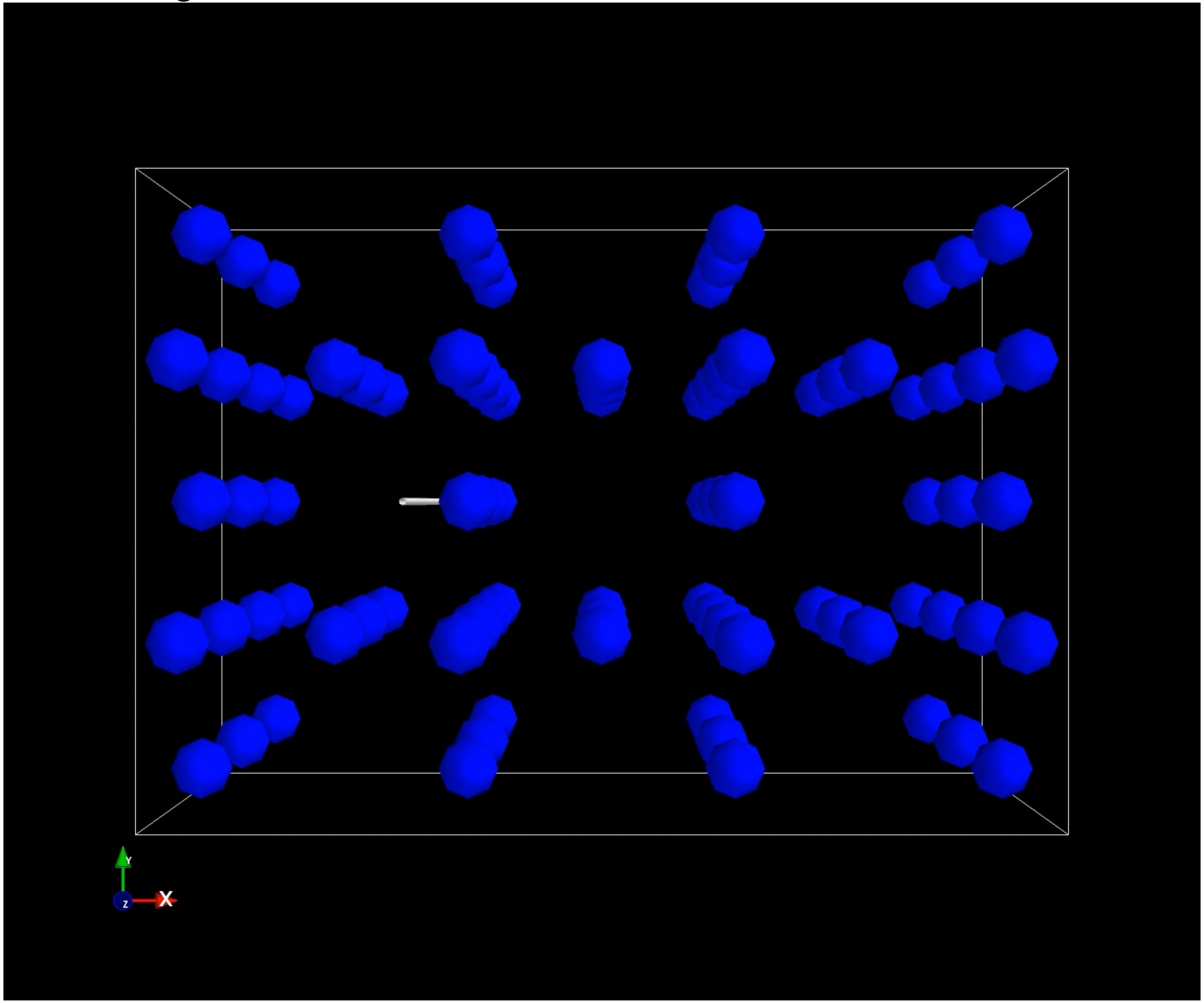
$\longrightarrow$ $V_{ext} < v_{crit}$

Put all elements together...

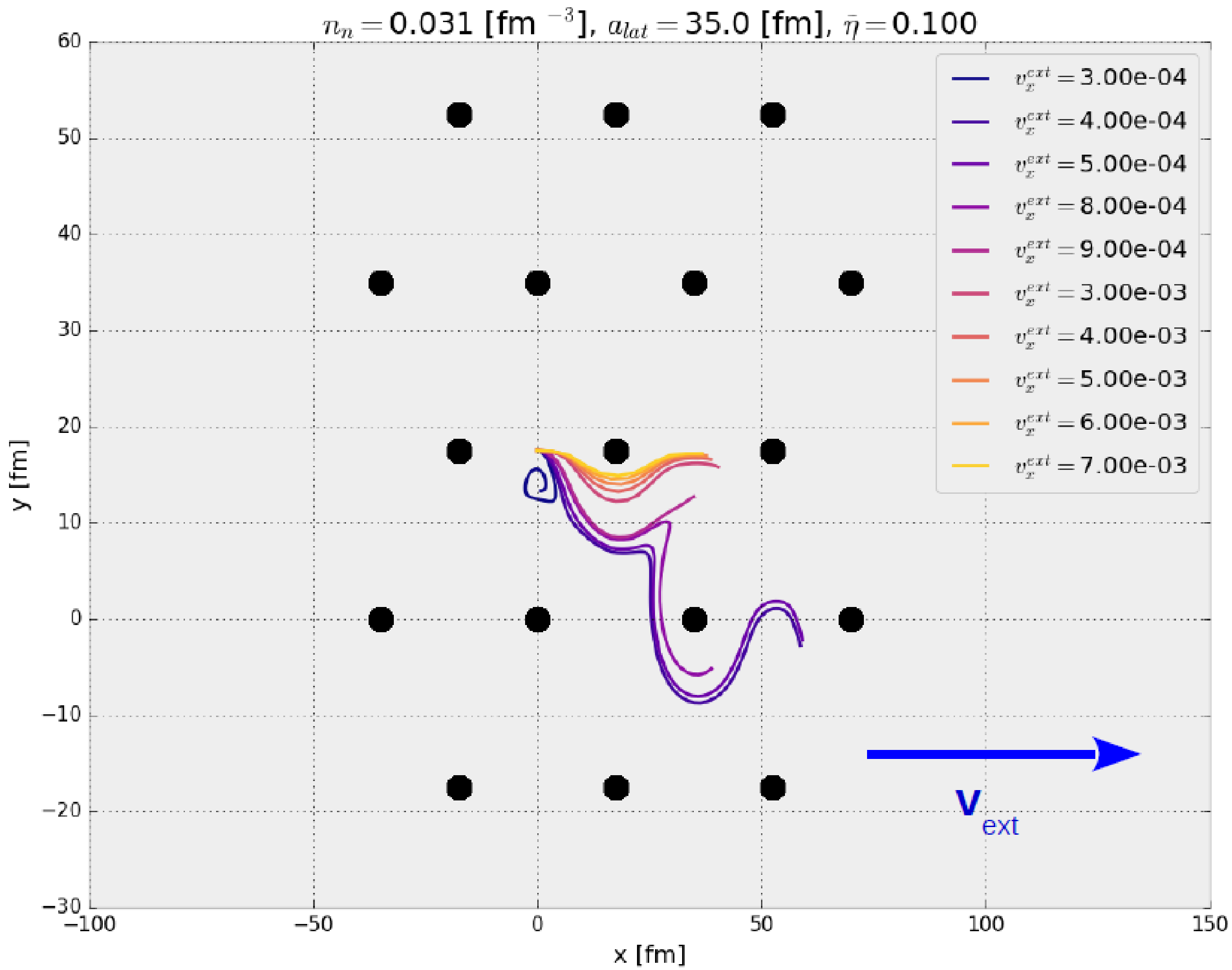


$V_{ext} > v_{crit}$

Put all elements together...



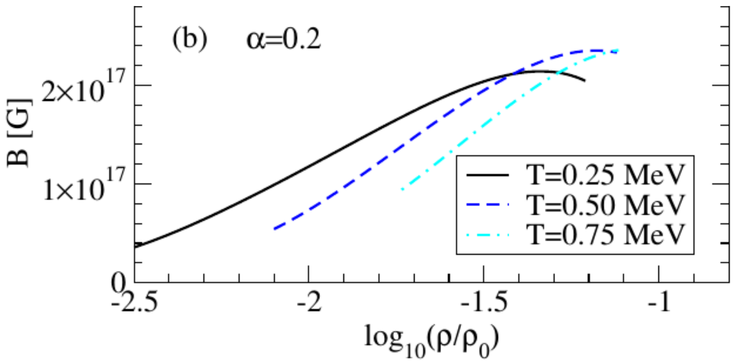
$V_{ext} \approx v_{crit}$



From K. Kobuszewski talk, POLNS18, 26-28 March 2018, Warsaw

Magnetic field and the vortex core structure

Fig. From: Stein et.al., Phys. Rev. C 93, 015802 (2016)



$$\alpha = \frac{\rho_\uparrow - \rho_\downarrow}{\rho_\uparrow + \rho_\downarrow}$$

FIG. 15: (Color online) Magnetic field required to create a specified spin polarization as a function of the density for two polarization values $\alpha = 0.1$ (a) and 0.2 (b) and temperatures $T = 0.25$ (solid line), 0.5 (dashed line), and 0.75 (dash-dotted line).

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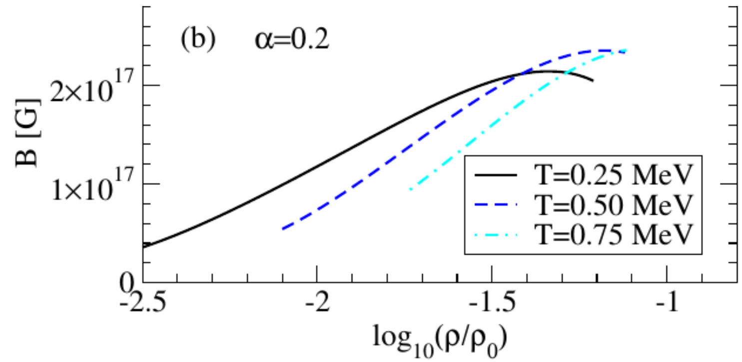


FIG. 15: (Color online) Magnetic field required to create a specified spin polarization as a function of the density for two polarization values $\alpha = 0.1$ (a) and 0.2 (b) and temperatures $T = 0.25$ (solid line), 0.5 (dashed line), and 0.75 (dash-dotted line).

Expectation:

→ …

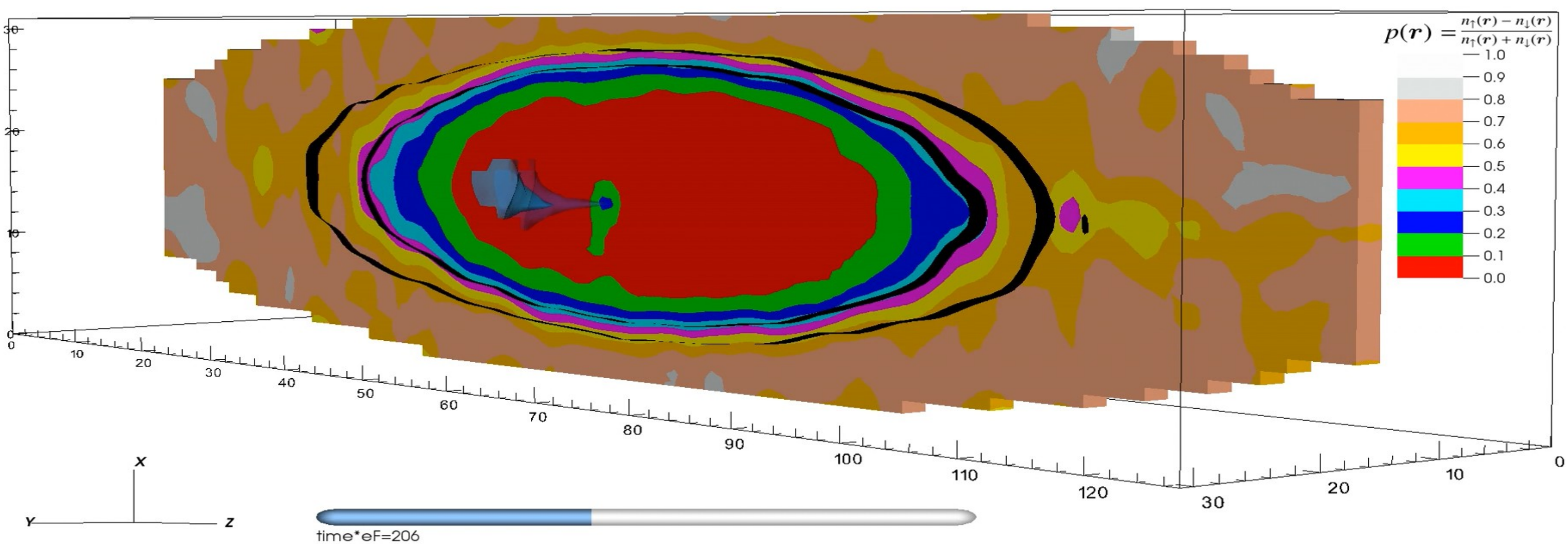
→ ...

→ short range physics – affected by the magnetic filed

$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$

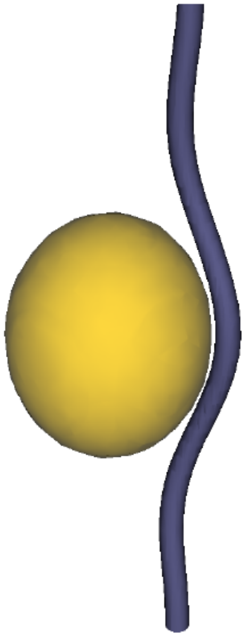
$$\alpha = \frac{\rho_\uparrow - \rho_\downarrow}{\rho_\uparrow + \rho_\downarrow}$$

$$N_\uparrow = 304,\ N_\downarrow = 202,\ P = 20\%$$



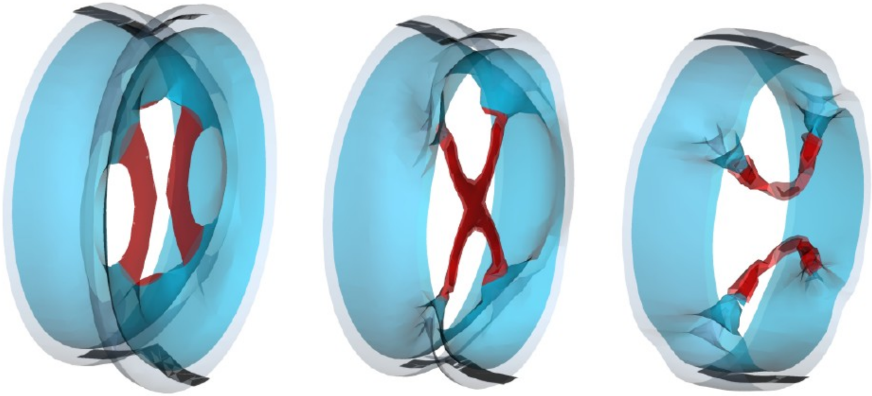
Examples

→ Vortex tension ($T=\Delta E_V/\Delta L$)



→ Short distance vortex-nucleus interaction

→ Short distance vortex-vortex interaction



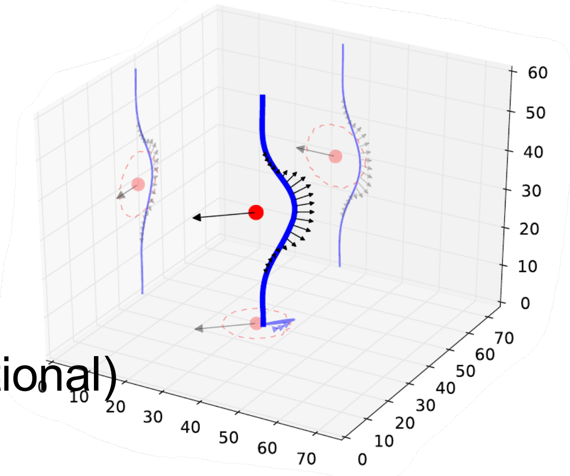
→ Dissipation process

What is expected magnetic field in the crust?

CONCLUSIONS

- DFT – route for unified description of static and dynamic properties of large Fermi systems

- TDDFT can be used as a source of microscopic input for pulsar glitch models
  - *We have defined information propagation scheme: TDDFT → VFM → Hydro*
  - *Target: indentify dominant source of pinning (crust or core)*



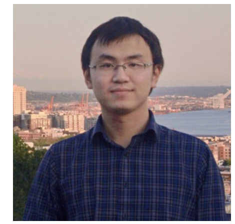
- We plan to execute campaign of systematic simulations in 2019-2020 (scan over densities with modern BSk functional)

- TDDFT has also been applied to other systems
  - *Dynamics in ultracold atoms* (vortex dynamics, quantum turbulence, shock waves...)
  - *Dynamics of nuclear systems* (fission, nuclear reactions, relativistic coulomb excitation...)

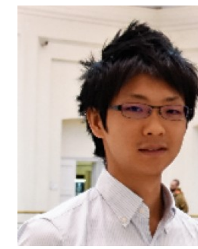
Warsaw University of Technology (Poland):
P. Magierski, G. Wlazłowski, B. Tuzemen, J. Olenicz, K. Kobuszewski



University of Washington (USA):
A. Bulgac, S. Jin

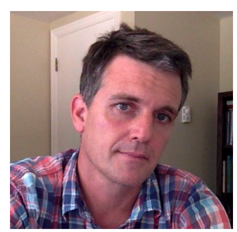
Niigata University (Japan):
K. Sekizawa



Washington State University (USA):
M. Forbes, R. Corbin

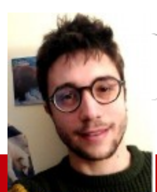
Los Alamos National Lab (USA):
I. Stetcu

Pacific Northwest National Lab (USA):
K. Roche (HPC)



Lawrence Livermore National Lab (USA):
N. Schunck

Nicolaus Copernicus Astronomical Center (Poland):
B. Haskell, M. Antonelli, V. Khomenko

PHAROS
THE MULTI-MESSENGER PHYSICS AND ASTROPHYSICS OF NEUTRON STARS

http://www.pharos.ice.csic.es/
(cost action CA16214)

ICM (Poland):
M. Marchwiany (HPC)

PRACE

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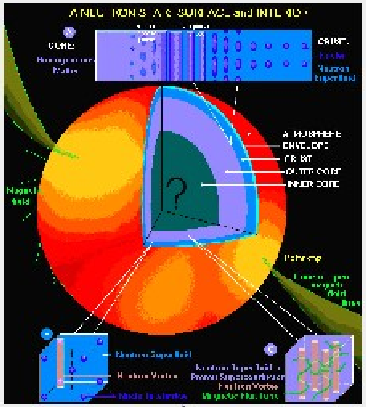
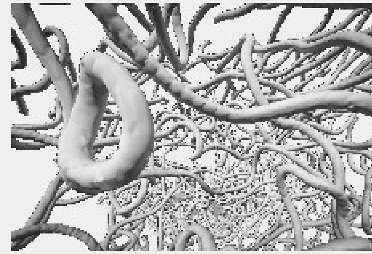
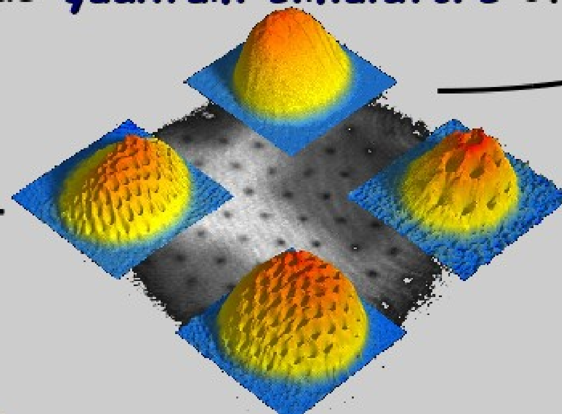
Superfluid effects in neutron stars (glitches, turbulence)



Quantum turbulence

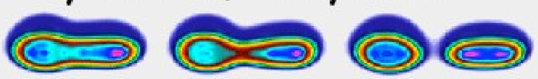


Ultracold fermionic gases as *quantum simulators* of...

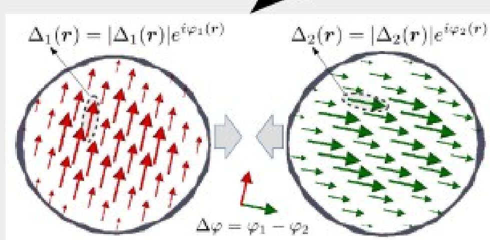


Supercomputing

Impact of superfluidity on fission dynamics of heavy nucleus



Collisions of two superfluid nuclei



Open call for Postdoc and PhD student position.

Thank you

Contact: gabrielw@if.pw.edu.pl

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Kohn-Sham "trick":

> Formally rigorous way of approaching any **interacting problem** by **mapping** it exactly to a much easier-to-solve **noninteracting system**.

easy, if Energy Density Functional (EDF) is known...

$$|\Psi\rangle \qquad\qquad |\Phi\rangle$$

interacting system — non-interacting system

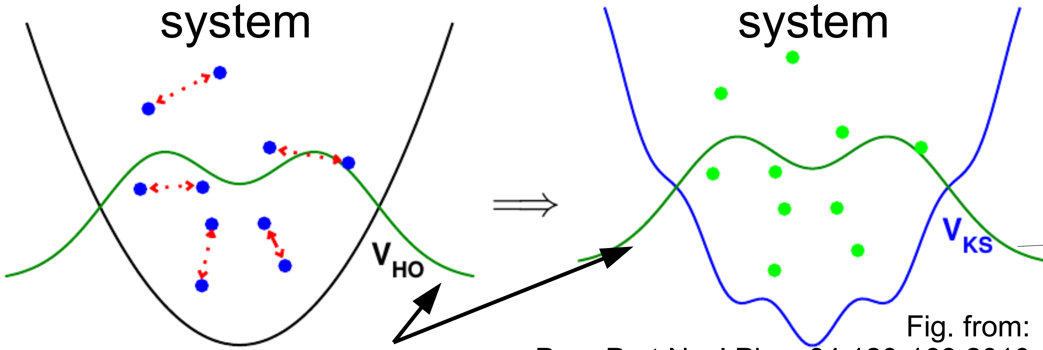


Fig. from: Prog.Part.Nucl.Phys.64:120-168,2010

Both systems described by the same density ρ

$$v_{KS} = \frac{\delta E_{\rm int}}{\delta \rho} + U_{\rm ext}$$

DFT theorem says (Hohenberg & Kohn):

$$\langle\Psi|\hat{H}_{\rm int} + \hat{U}_{\rm ext}|\Psi\rangle = E = E[\rho]$$

"Universal" part — External field

$$E[\rho] = E_{\rm int}[\rho] + \int \rho(\boldsymbol{r},t)U_{\rm ext}(\boldsymbol{r},t)\,d\boldsymbol{r}$$

Equality of energies for both systems.

Moreover, Kohn-Sham method:

$$\langle\Psi|\hat{O}_{1B}|\Psi\rangle = O_{1B} = \langle\Phi|\hat{O}_{1B}|\Phi\rangle$$

HF equations

What about superfluids?

Normal system

$$\begin{cases} [-\nabla^2/2m + v_{\mathrm{KS}}(\mathbf{x})]\phi_i(\mathbf{x}) = \varepsilon_i\phi_i(\mathbf{x}) \\ v_{KS} = \dfrac{\delta E_{\mathrm{int}}}{\delta\rho} + U_{\mathrm{ext}} \end{cases}$$

pairing (anomalous) density

$$E_{\mathrm{int}}[\rho,\ldots] \rightarrow E_{\mathrm{int}}[\rho,\nu,\ldots]$$

$$\begin{cases} [h(\mathbf{r}) - \mu]u_k(\mathbf{r}) + \Delta(\mathbf{r})v_k(\mathbf{r}) = E_k u_k(\mathbf{r}), \\ \Delta^*(\mathbf{r})u_k(\mathbf{r}) - [h(\mathbf{r}) - \mu]v_k(\mathbf{r}) = E_k v_k(\mathbf{r}), \end{cases}$$

HFB equations

$$h(\mathbf{r}) = -\nabla^2/2m + v_{\mathrm{KS}}(\mathbf{x})$$

$$\Delta = -\frac{\delta E_{\mathrm{int}}}{\delta\nu^*}$$

$$E_k \rightarrow i\hbar\frac{\partial}{\partial t}$$

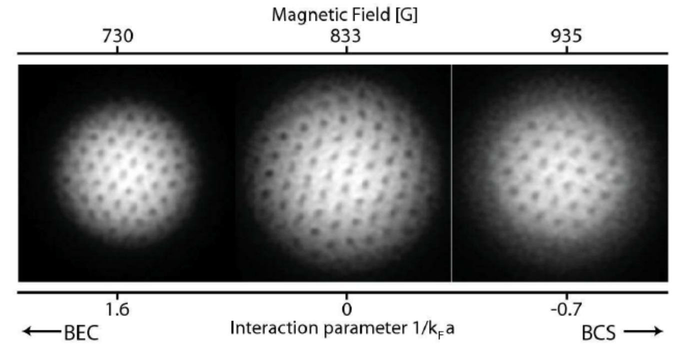


FIG. 36 Vortex lattice in a rotating gas of ^{6}Li precisely at the Feshbach resonance and on the BEC and BCS side. Reprinted with permission from Zwierlein *et al.* (2005).

Mapping **on system of non-interacting quasiparticles**

Density-Functional Theory for Time-Dependent Systems

Erich Runge and E. K. U. Gross

Phys. Rev. Lett. **52**, 997 – Published 19 March 1984

Solving time-dependent problem for superfluids...

The real-time dynamics is given by equations, which are formally equivalent to the Time-Dependent HFB (TDHFB) or Time-Dependent Bogolubov-de Gennes (TDBdG) equations

Spatial derivatives hidden here...

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}u_{n,\uparrow}(\boldsymbol{r},t)\\ u_{n,\downarrow}(\boldsymbol{r},t)\\ v_{n,\uparrow}(\boldsymbol{r},t)\\ v_{n,\downarrow}(\boldsymbol{r},t)\end{pmatrix}=\begin{pmatrix}h_{\uparrow,\uparrow}(\boldsymbol{r},t) & h_{\uparrow,\downarrow}(\boldsymbol{r},t) & 0 & \Delta(\boldsymbol{r},t)\\ h_{\downarrow,\uparrow}(\boldsymbol{r},t) & h_{\downarrow,\downarrow}(\boldsymbol{r},t) & -\Delta(\boldsymbol{r},t) & 0\\ 0 & -\Delta^*(\boldsymbol{r},t) & -h^*_{\uparrow,\uparrow}(\boldsymbol{r},t) & -h^*_{\uparrow,\downarrow}(\boldsymbol{r},t)\\ \Delta^*(\boldsymbol{r},t) & 0 & -h^*_{\downarrow,\uparrow}(\boldsymbol{r},t) & -h^*_{\downarrow,\downarrow}(\boldsymbol{r},t)\end{pmatrix}\begin{pmatrix}u_{n,\uparrow}(\boldsymbol{r},t)\\ u_{n,\downarrow}(\boldsymbol{r},t)\\ v_{n,\uparrow}(\boldsymbol{r},t)\\ v_{n,\downarrow}(\boldsymbol{r},t)\end{pmatrix}$$

We explicitly track fermionic degrees of freedom!

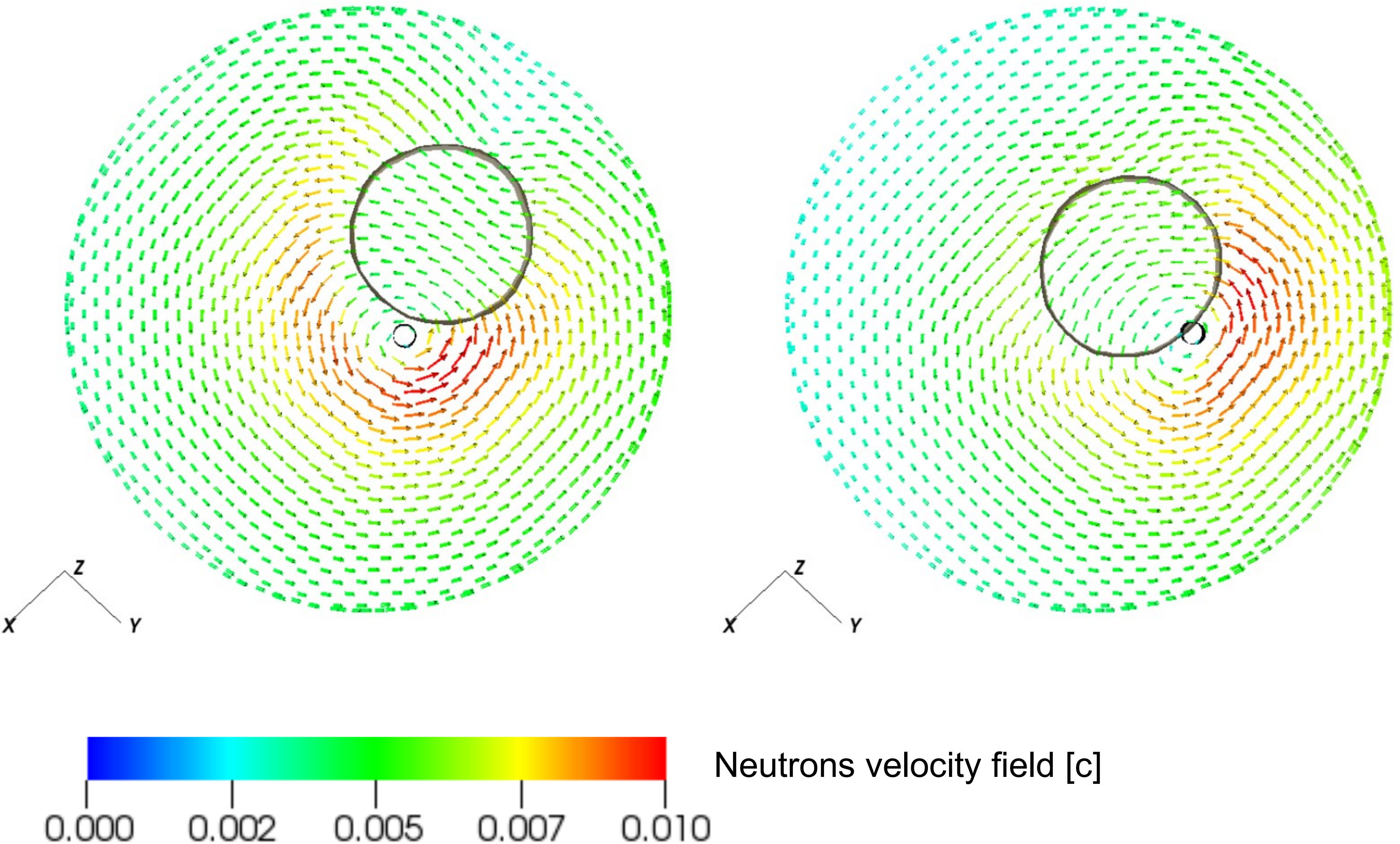
where *h* and Δ depends on "densities":

$$n_\sigma(\boldsymbol{r},t)=\sum_{E_n<E_c}|v_{n,\sigma}(\boldsymbol{r},t)|^2,\qquad \tau_\sigma(\boldsymbol{r},t)=\sum_{E_n<E_c}|\nabla v_{n,\sigma}(\boldsymbol{r},t)|^2,$$

$$\nu(\boldsymbol{r},t)=\sum_{E_n<E_c}u_{n,\uparrow}(\boldsymbol{r},t)v^*_{n,\downarrow}(\boldsymbol{r},t),\qquad \boldsymbol{j}_\sigma(\boldsymbol{r},t)=\sum_{E_n<E_c}\mathrm{Im}[v^*_{n,\sigma}(\boldsymbol{r},t)\nabla v_{n,\sigma}(\boldsymbol{r},t)],$$

a lot of nonlinear coupled 3D Partial Differential Equations
(in practice 10^5 - 10^6)

disturbance of velocity field by impurity
→ origin of effective vortex-nucleus interaction



We performed 3D, dynamical simulations by TDDFT with superfluidity

- TDSLDA equations (similar to TDHFB, TD-BdG)

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_k \\ v_k \end{pmatrix} = \begin{pmatrix} h & \Delta \\ \Delta^* & -h \end{pmatrix}\begin{pmatrix} u_k \\ v_k \end{pmatrix}$$

- Potentials

$$h = \frac{\delta\mathcal{E}}{\delta n}, \quad \Delta = \frac{\delta\mathcal{E}}{\delta\nu^*}$$

$$n(\boldsymbol{r}) = \sum_{0<E_k<E_c} |v_k(\boldsymbol{r})|^2$$

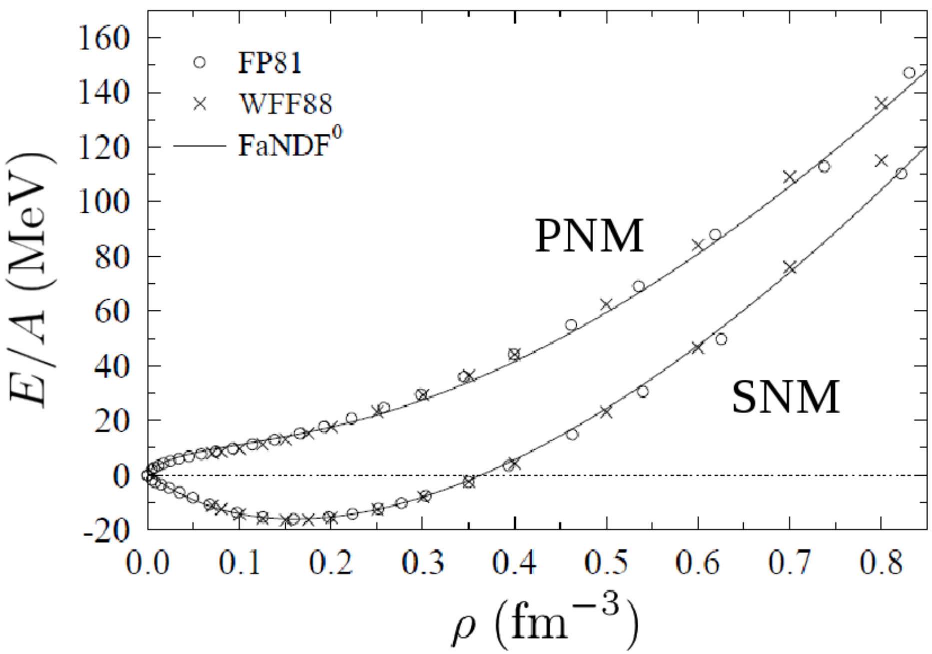
$$\nu(\boldsymbol{r}) = \sum_{0<E_k<E_c} u_k(\boldsymbol{r})v_k^*(\boldsymbol{r})$$

- Energy density functional (EDF)

$$\mathcal{E} = \mathcal{E}_0 + \mathcal{E}_{\text{pair}}$$

$\mathcal{E}_0$: Fayans EDF (FaNDF^0) w/o LS

S.A. Fayans, JETP Letters 68, 169 (1998);



FP81: B. Friedman and V. R. Pandharipande, Nucl. Phys. A 361, 502 (1981)

WFF88: R. B. Wiringa, V. Fiks, and A. Fabrocini, Phys. Rev. C 38, 1010 (1988).

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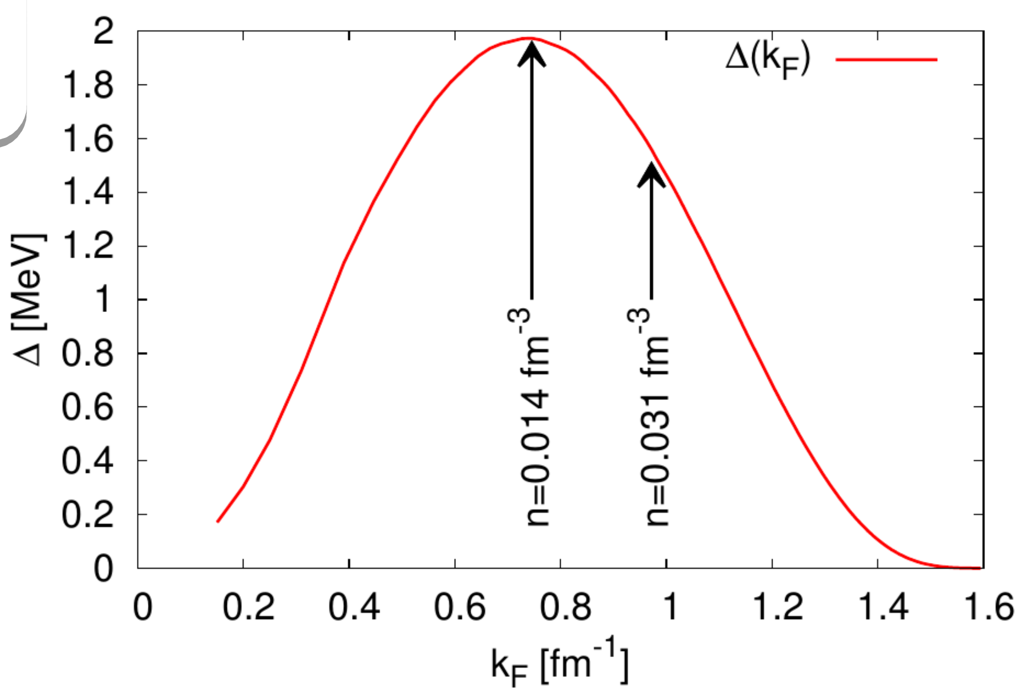
$$\nu(\boldsymbol{r}) = \sum_{0<E_k<E_c} u_k(\boldsymbol{r})v_k^*(\boldsymbol{r})$$

- Energy density functional (EDF)

$$\mathcal{E} = \mathcal{E}_0 + \mathcal{E}_{\text{pair}}$$

$$\mathcal{E}_{\text{pair}}(\boldsymbol{r}) = g(n(\boldsymbol{r}))\left[|\nu_n(\boldsymbol{r})|^2 + |\nu_p(\boldsymbol{r})|^2\right]$$

The coupling constant g is chosen to reproduce the neutron pairing gap in pure neutron matter.



Vortex-impurity interaction – present status

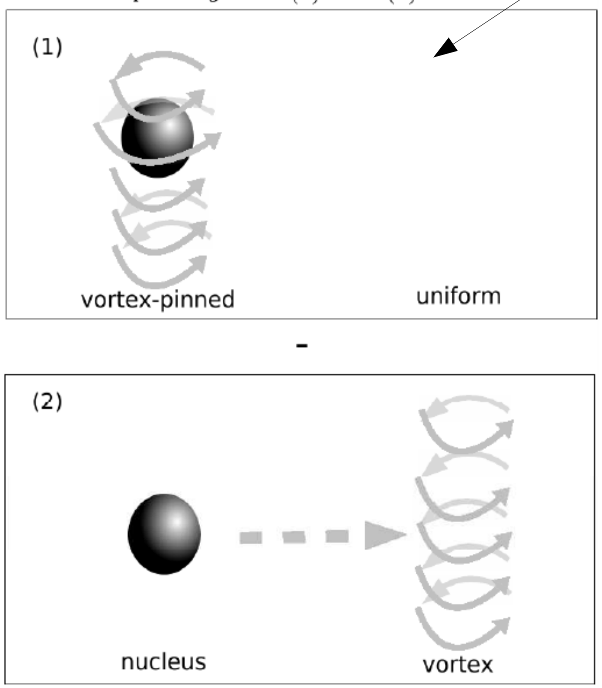
only calculations of *pinning energy...*

~1 MeV ~10,000 MeV

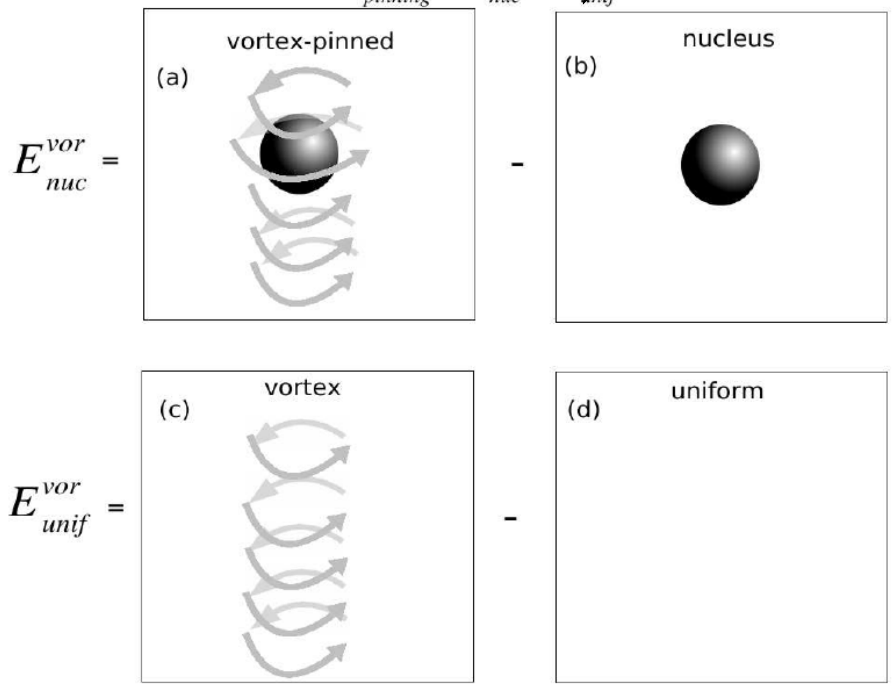
only symmetric configurations have been considered with energies of order 10,000 MeV

Calculations for fixed chemical potential + correction $\Delta E = \mu \Delta N$

$E_{pinning} = E_{(1)} - E_{(2)}$



$E_{pinning} = E^{vor}_{nuc} - E^{vor}_{unif}$

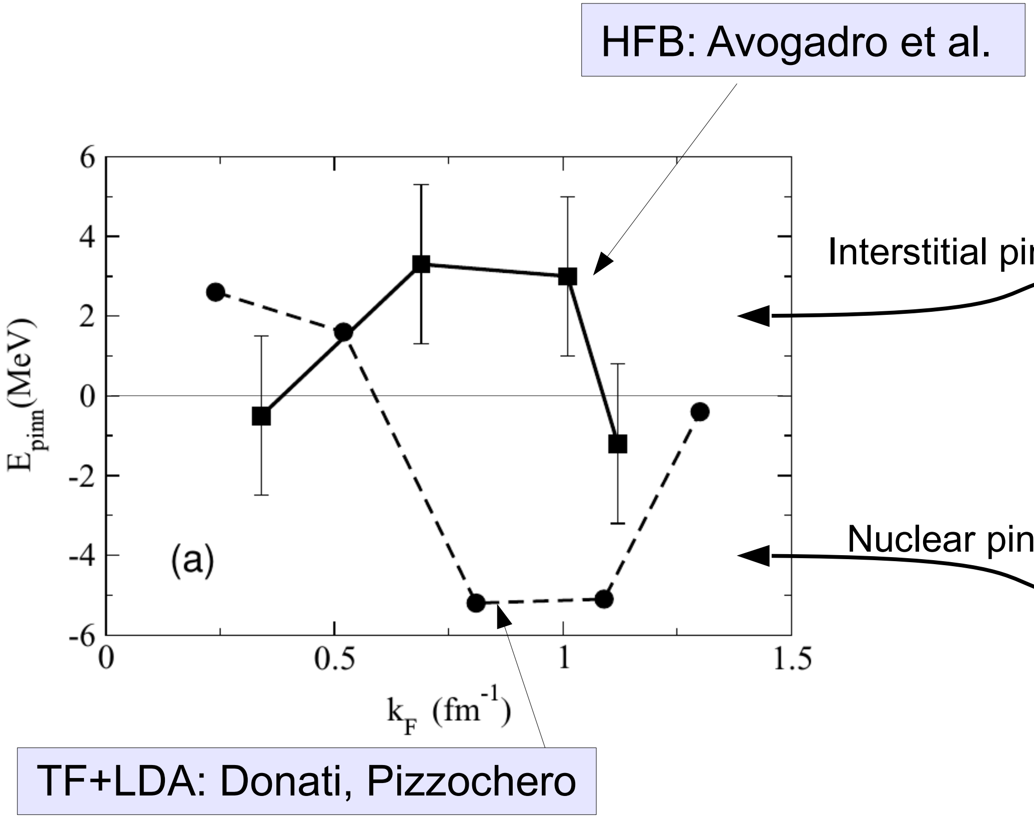


Fixed chemical potential or neutron background density?

Volume: tube of radius 30fm and height of 40fm

SLy4 $n_\infty = 0.026\ \mathrm{fm}^{-3}$		
Pinned	8.55	12956.57
Nucleus	8.50	12954.02
Vortex	8.55	13712.91
Uniform	8.50	13617.05
[MeV]	Chem.pot	energy

Figures and numbers from:
P. Avogadro, F. Barranco, R.A. Broglia, E. Vigezzi Nucl. Phys. A811, 378-412 (2008)



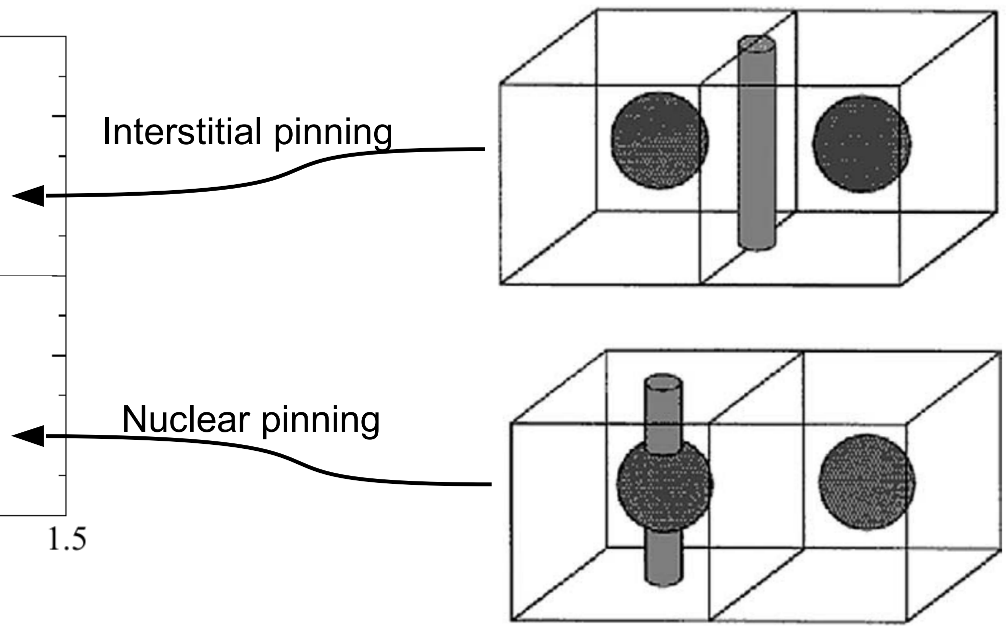
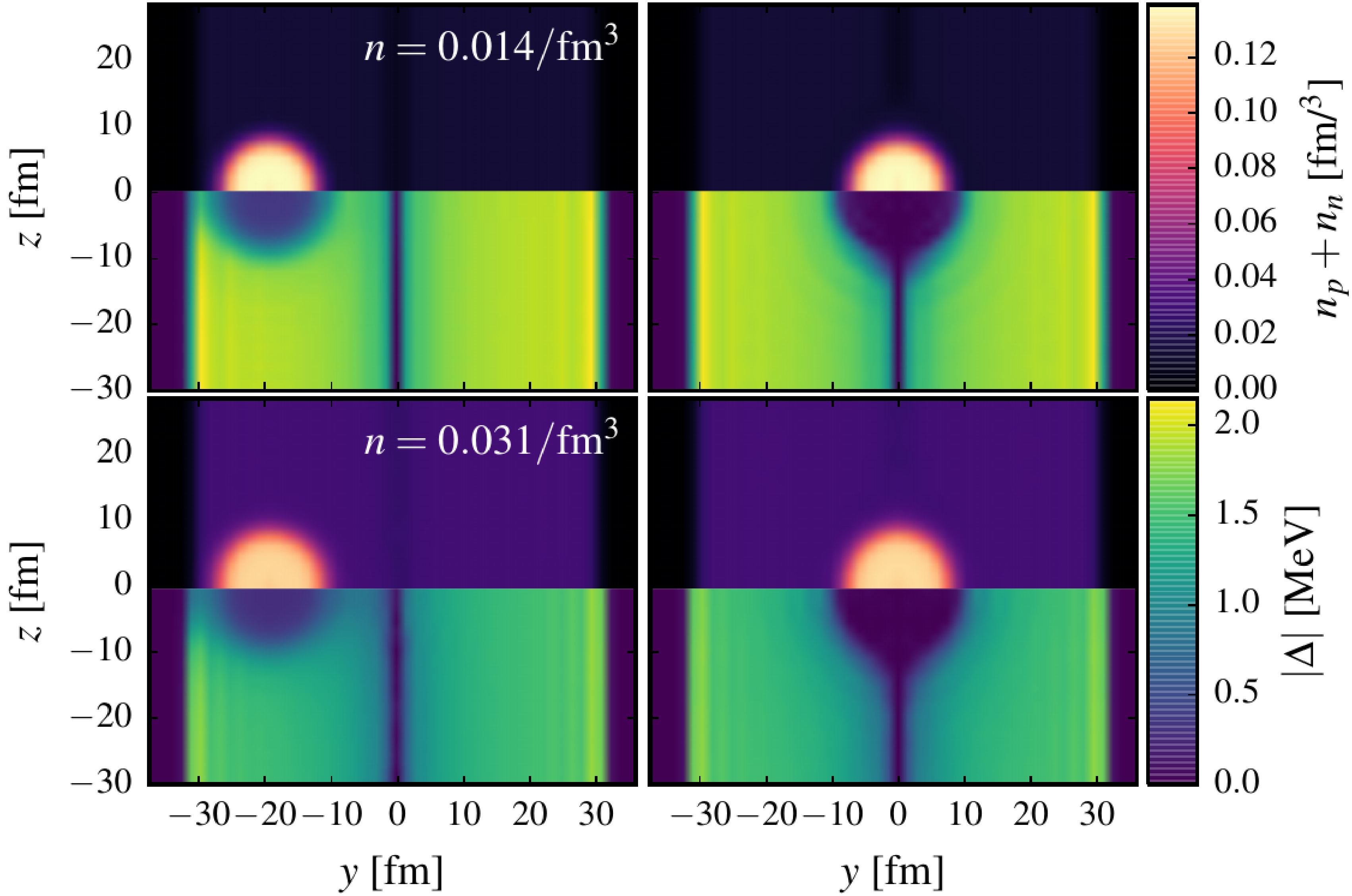


Fig. from: P. Avogadro et al.,
Phys. Rev. C 75, 012805(R) (2007)

Figs from: P. Donati et al.,
Nuclear Physics A 742 (2004) 363



Static solutions: $E_{unpinned}$ < E_{pinned}

Dragging force

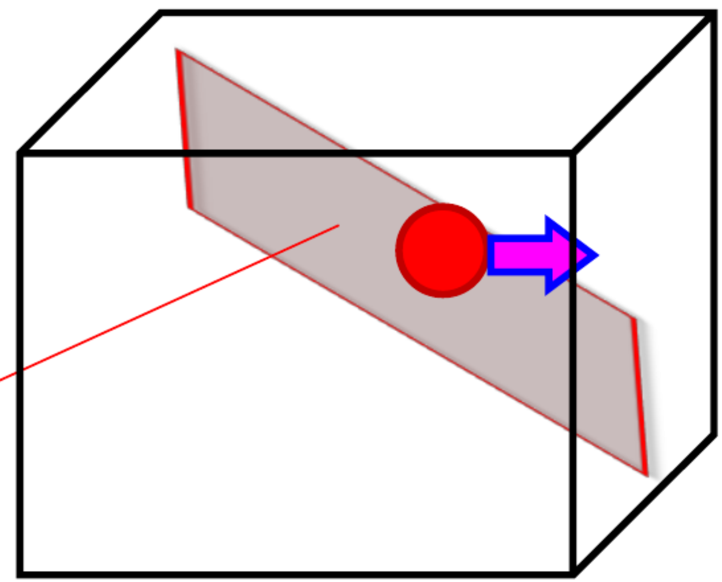
external time-dependent potential couples only to protons and it is constant in space.

Dragging speed:

$$v_0 = 0.001c \ll v_{\rm crit}$$

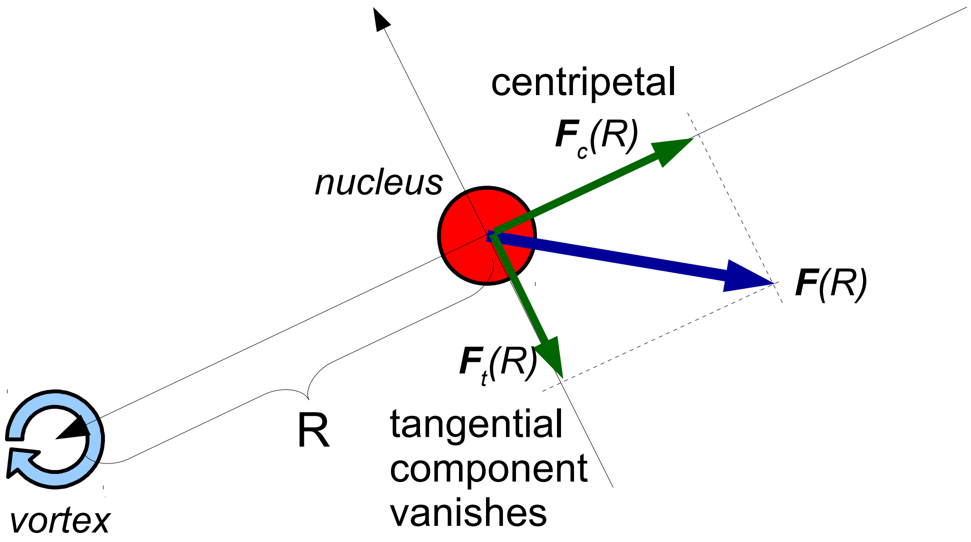
$$U_{\rm ext}(\boldsymbol{r}, t) = -\frac{1}{Z}\boldsymbol{F}_{\rm ext}(t) \cdot \boldsymbol{r}$$

$$\frac{d\left\langle \hat{\boldsymbol{p}} \right\rangle}{dt} = -\left\langle \boldsymbol{\nabla} U_{\rm ext}(\boldsymbol{r}, t)\right\rangle = \boldsymbol{F}_{\rm ext}(t)$$



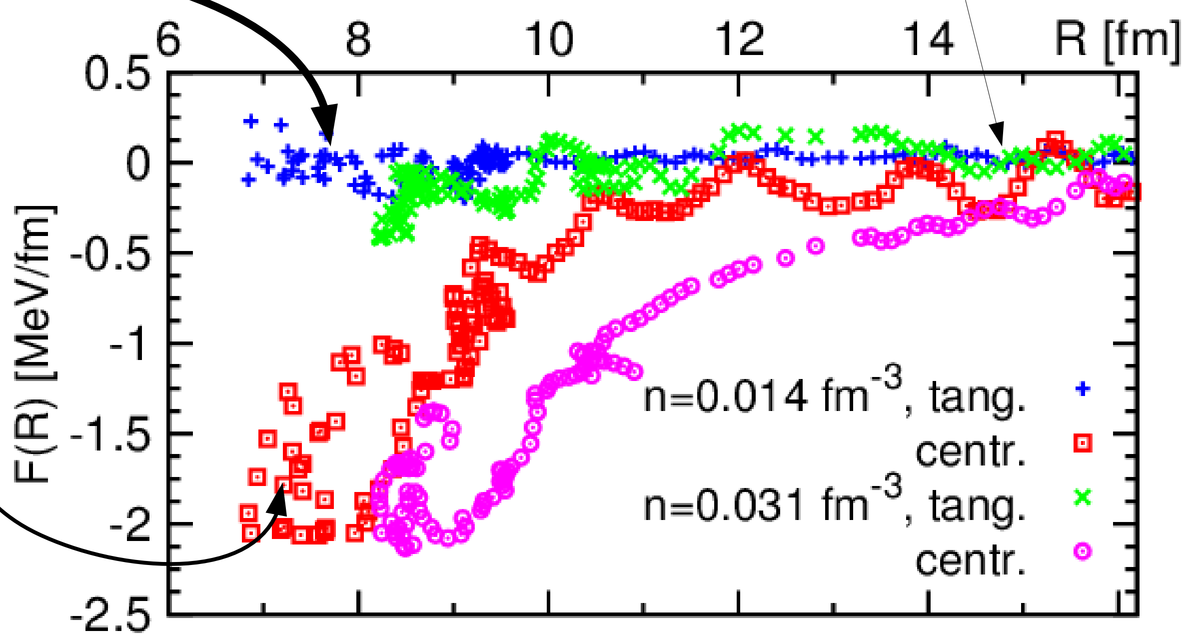
This force moves the center of mass of the protons together with those neutrons bound (entrained) in the nucleus without significantly modifying the internal structure of the nucleus and surrounding neutron medium

Force decomposition



Range: 10-15 fm

Force is **central**



At close separation the force is not a function of distance R only

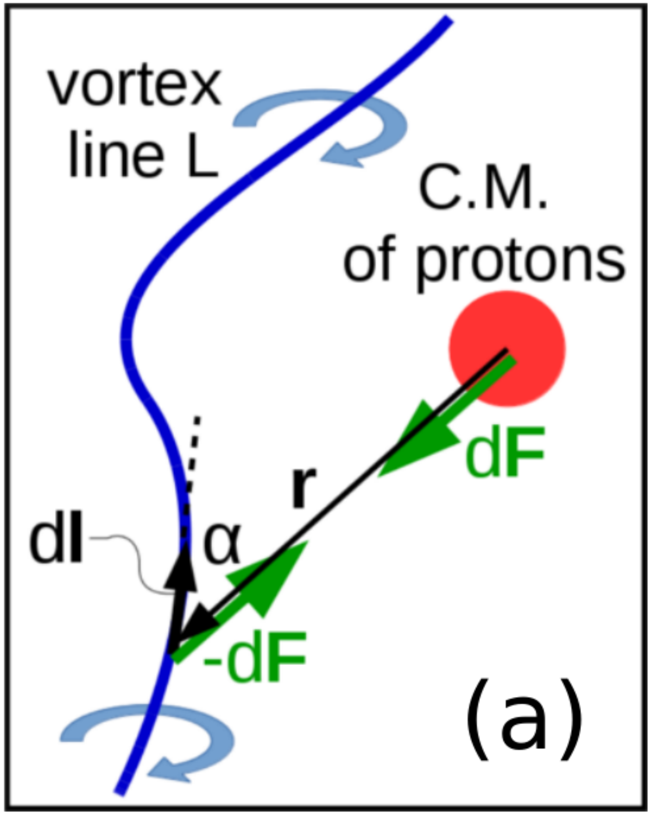
Force per unit length

Inspired by the vortex filament model (describes vortex-vortex Interaction)

$$\mathrm{d}\boldsymbol{F} = f(r)\sin\alpha\,\hat{\boldsymbol{r}}\,\mathrm{d}l \qquad \hat{\boldsymbol{r}} = \boldsymbol{r}/r$$

$$\boldsymbol{F} = \int_L \mathrm{d}\boldsymbol{F}$$

Total force acting on nucleus



Ansatz for f(r) - Pade approximant

$$f(r) = \frac{\sum_{k=0}^{n} a_k r^k}{1 + \sum_{k=1}^{n+3} b_k r^k}$$

$$f(r \to \infty) \propto 1/r^3$$

from irrotational and incompressible hydrodynamics

We minimize chi2 with respect of $\{a_k, b_k\}$

$$\chi_w^2 = \sum_{i=1}^{N} w(|\boldsymbol{F}_i|)\left(\boldsymbol{F}_i - \boldsymbol{F}_i^{(f)}(\{a_k, b_k\})\right)^2$$

"frames of movie"

Measured force

Predicted for given set $\{a_k, b_k\}$

$$\boldsymbol{F}_i^{(f)} = \int_{L_i} f(r; \{a_k, b_k\}) \sin\alpha\, \boldsymbol{e}_r\, dl$$