The improved Ginzburg-Landau technique and GW echoes from strange stars

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S.Carignano, F.Anzuini, O. Benhar, MM Phys.Rev. D97 (2018), 036009
MM and F. Tonelli, (2018), arXiv:1805.02278 [gr-qc] to appear in Phys.Rev. D

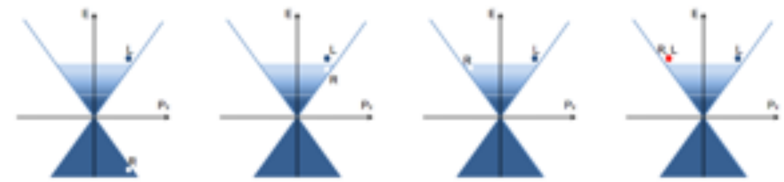
CSQCD VII
NY, June 12, 2018

Outline

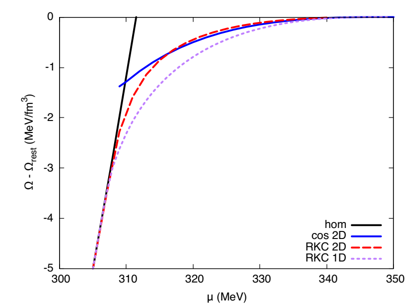
- **Background**

$$SU(3)_{\mathrm{c}} \times \underbrace{SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}}}_{\supset [U(1)_{\mathrm{e.m.}}]} \times U(1)_{\mathrm{B}}$$

- **Competing condensates**



- **Improved Ginzburg-Landau expansion**



Aside

- **Gravitational wave echoes from strange stars**
- **Conclusions**

BACKGROUND

Symmetries of QCD

Symmetries of the three flavor massless QCD Lagrangian

color gauge group

global chiral symmetry

global baryonic number

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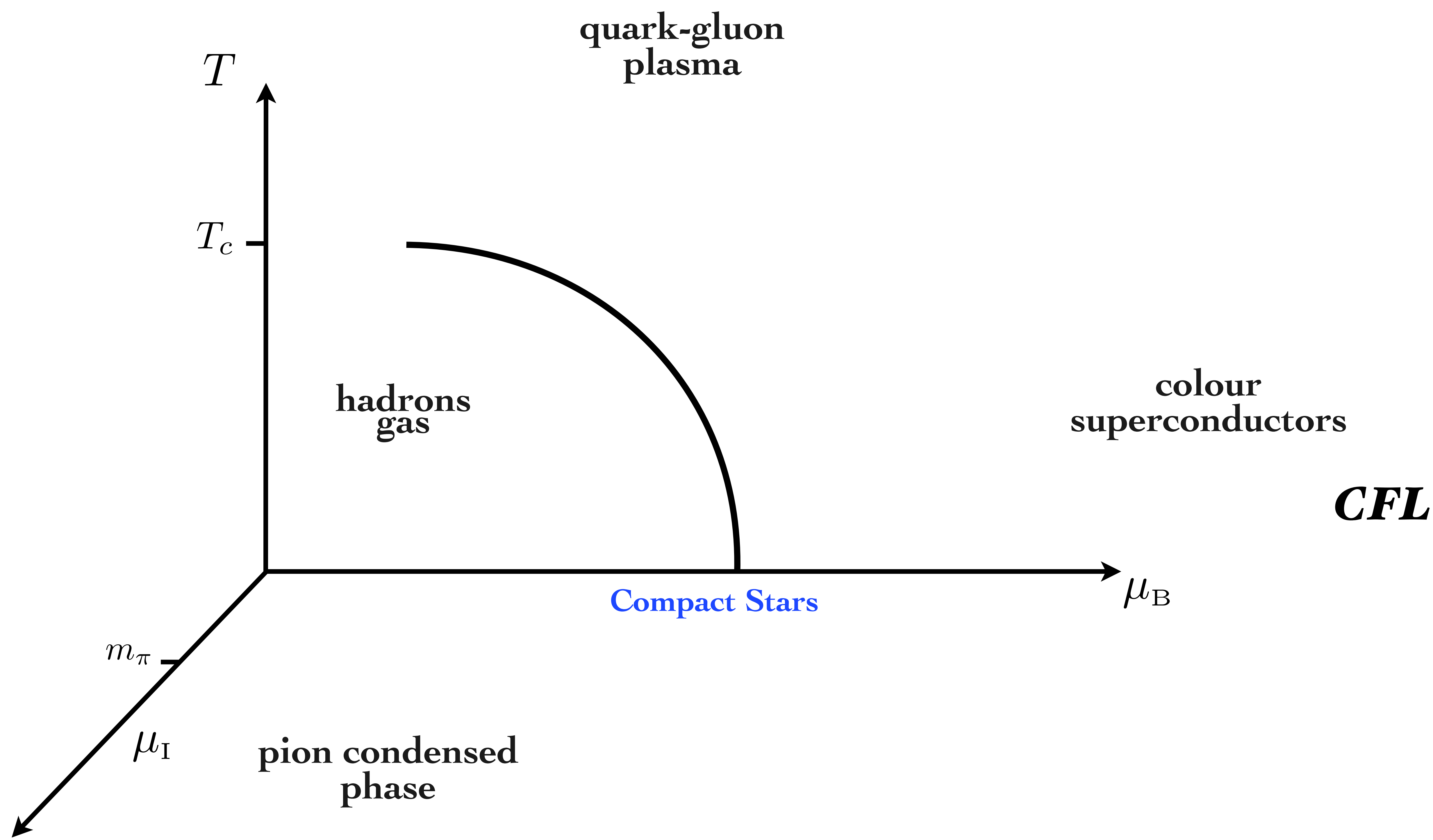
The ground state may have a lower symmetry because of quark condensates

$\langle\bar{\psi}\psi\rangle$ **Chiral condensate: Locks chiral rotations**

$\langle\bar{\psi}\sigma_2\gamma_5\psi\rangle$ **Pion condensate: Locks chiral rotations and breaks** $U(1)_{\mathrm{e.m.}}$

$\langle\psi C\gamma_5\psi\rangle$ **Diquark condensate: Breaks the gauge group and may lock chiral rotations**

Quark matter phase diagram



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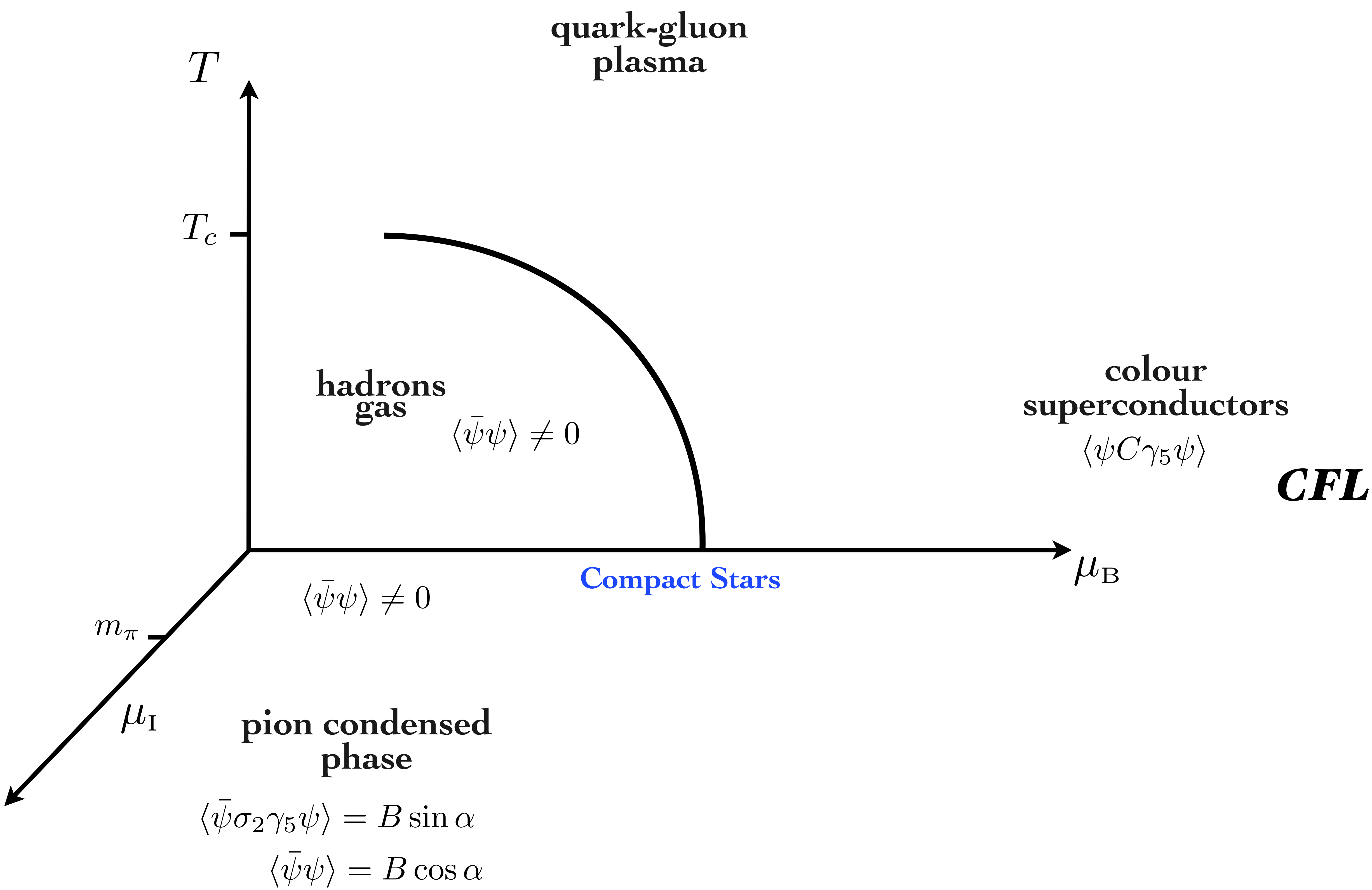
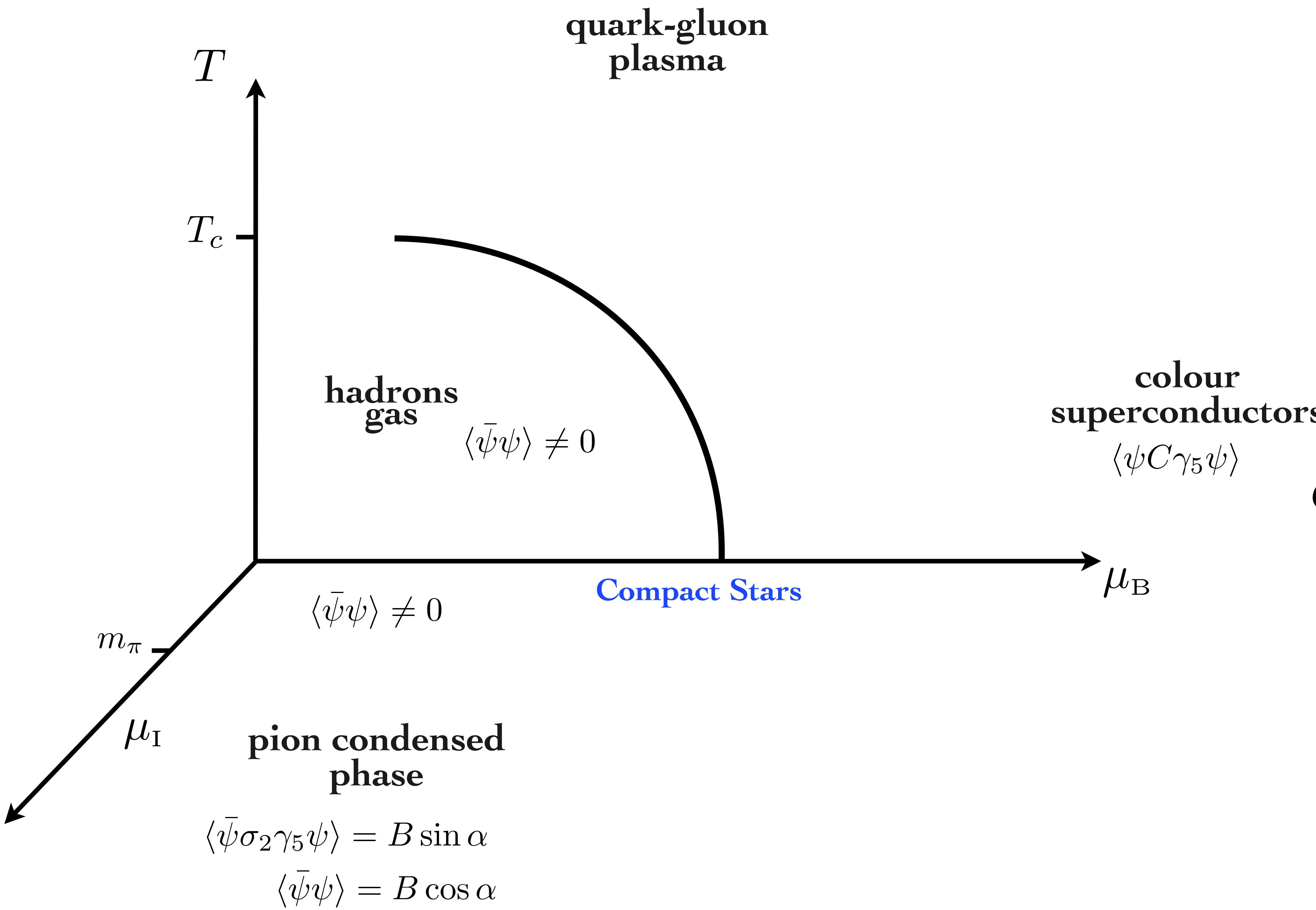


Quark matter phase diagram

SOME METHODS



CFL

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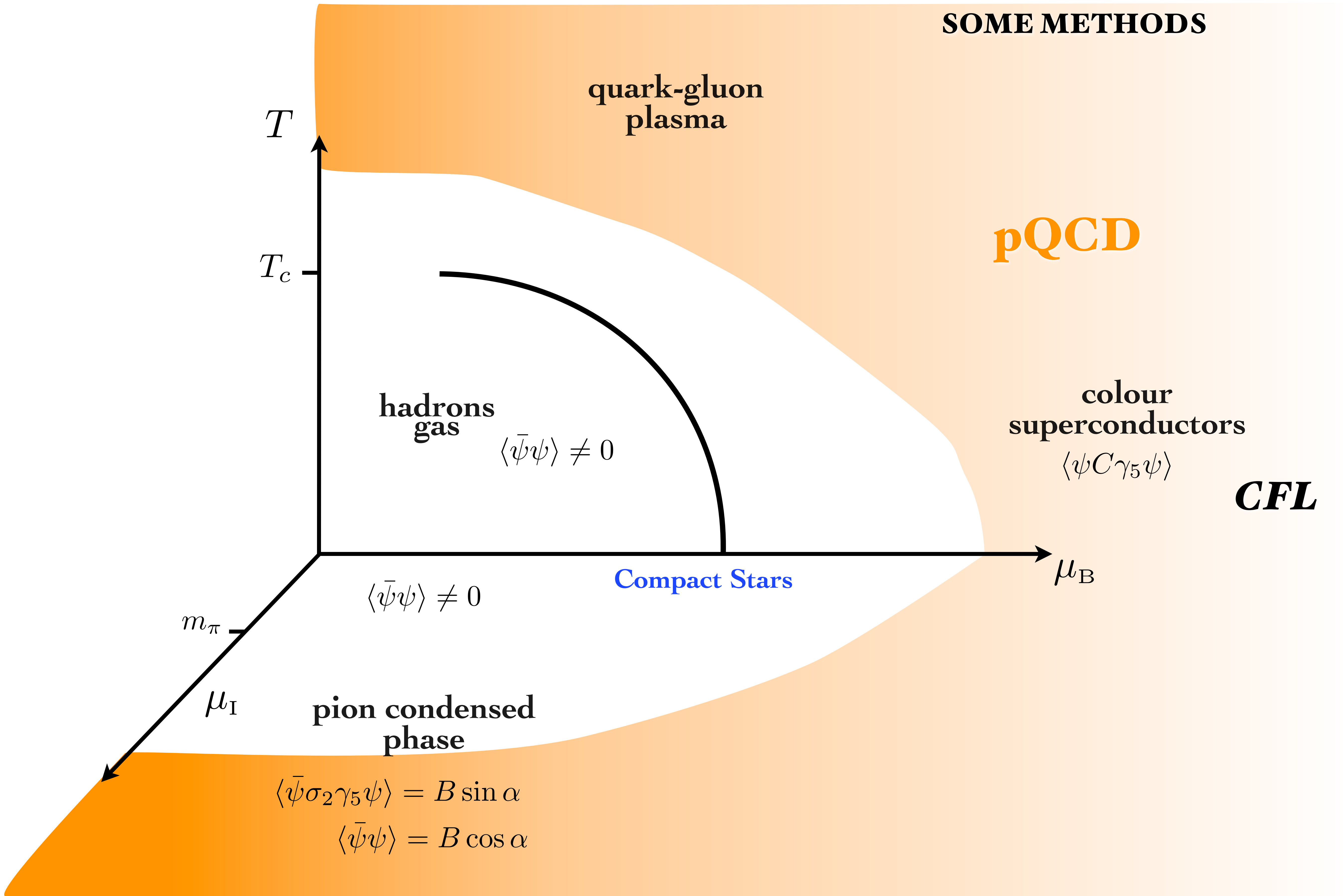

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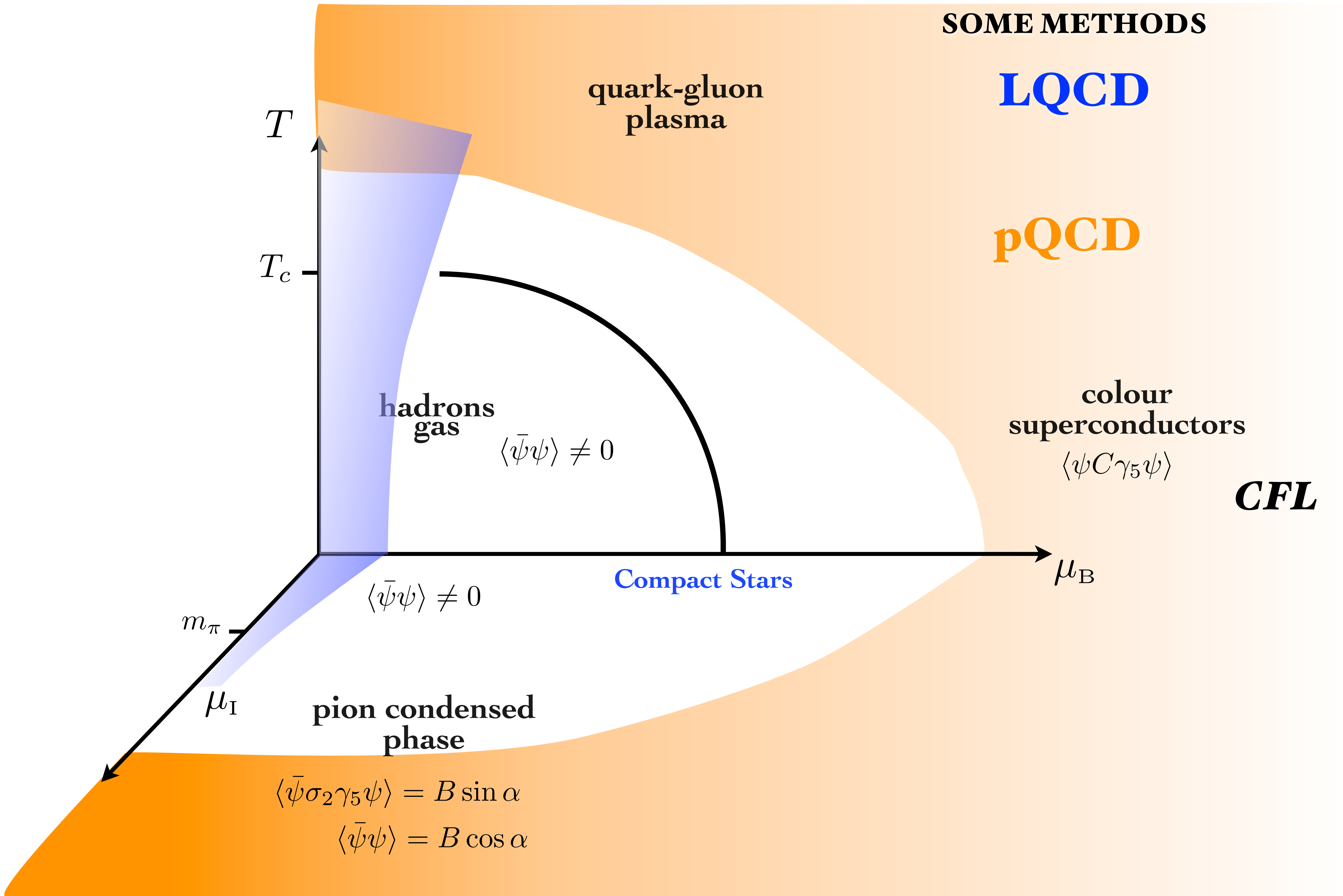

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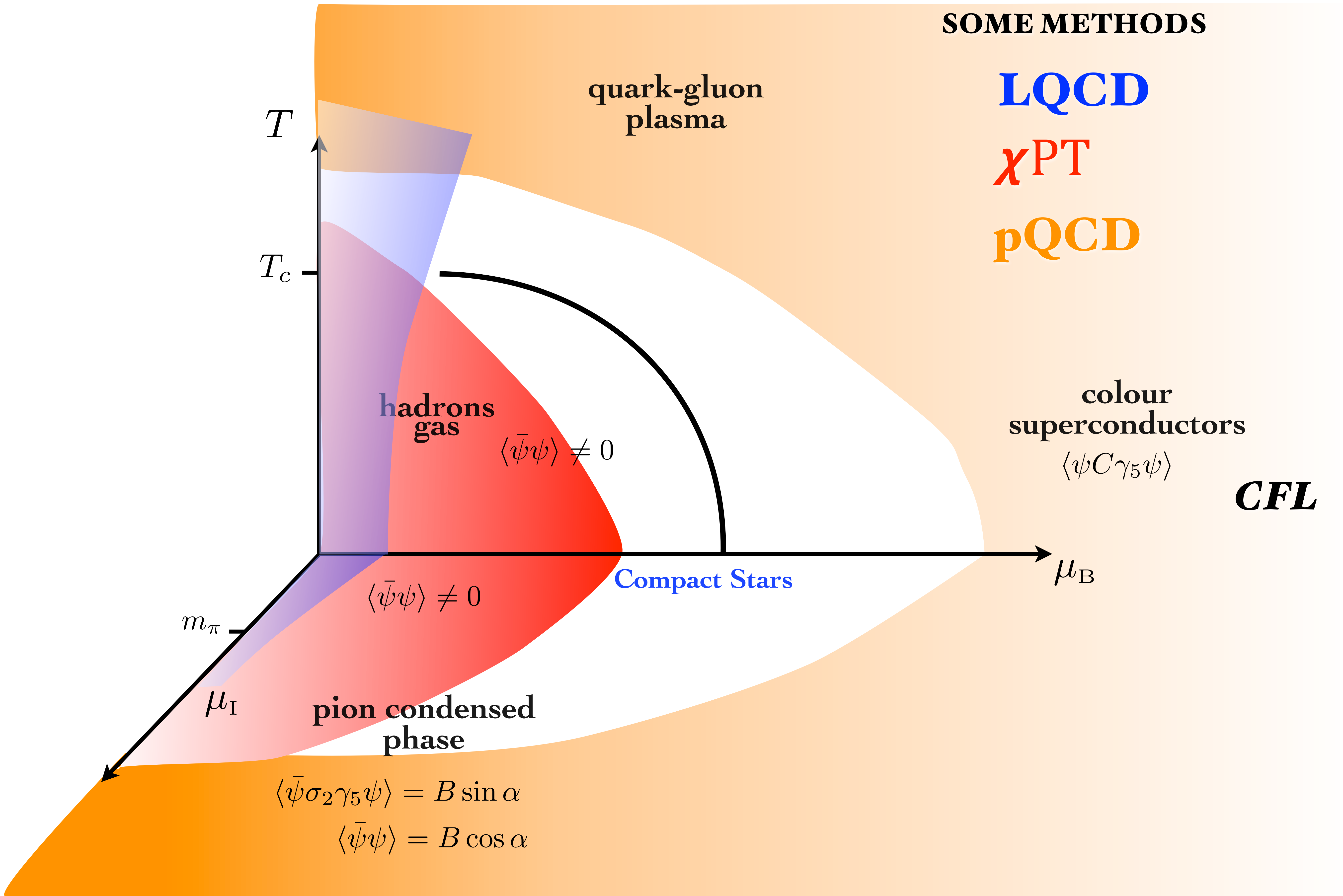

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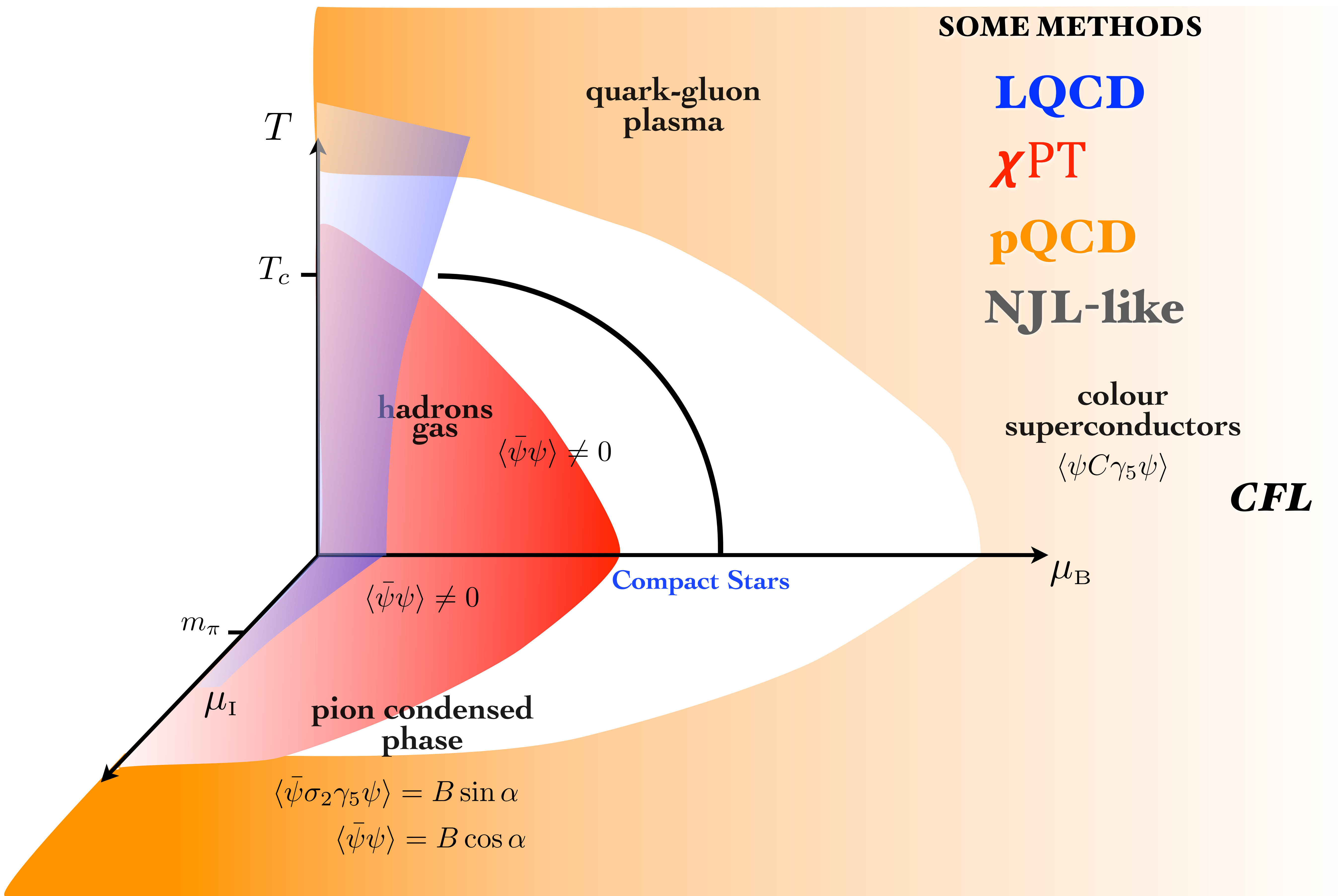

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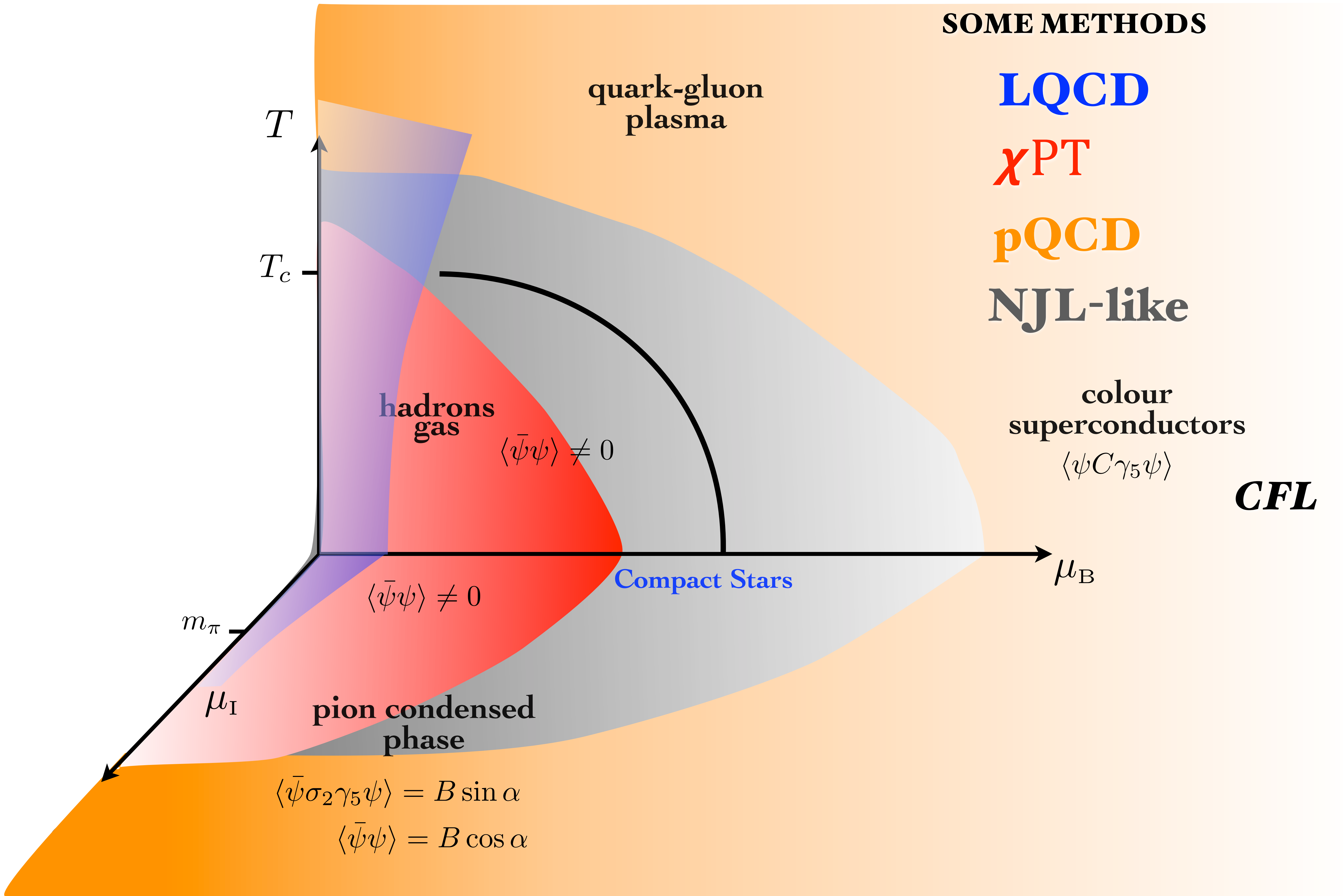
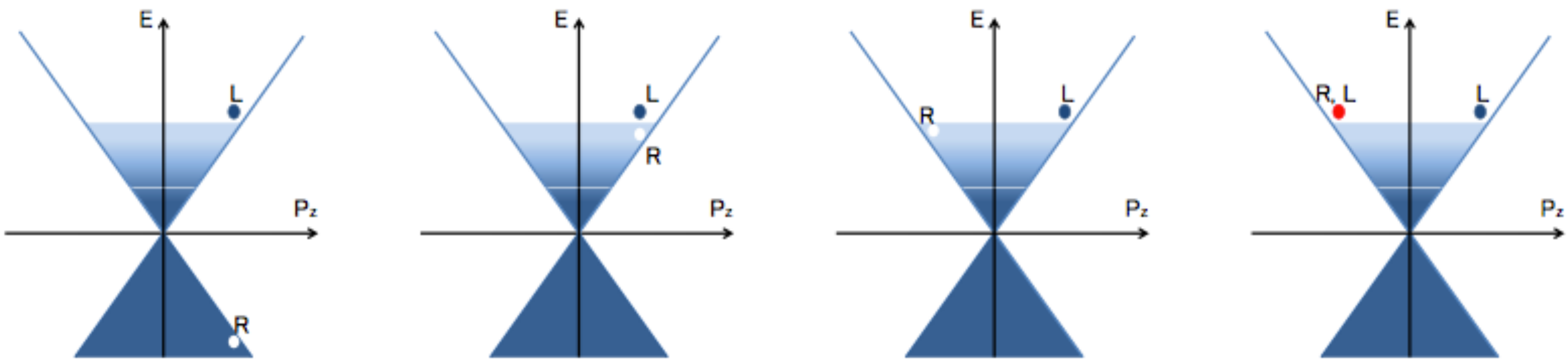


COMPETING CONDENSATES

Fight of condensates

Different kind of pairings

T. Kojo, Y. Hidaka, L. McLerran, R. D. Pisarski, Nucl. Phys. A 843 (2010) 37



quark-antiquark

quark-hole
(exciton-like)

quark-hole
(CDW)

quark-quark
color superconductor

Not obvious which of these is energetically favored

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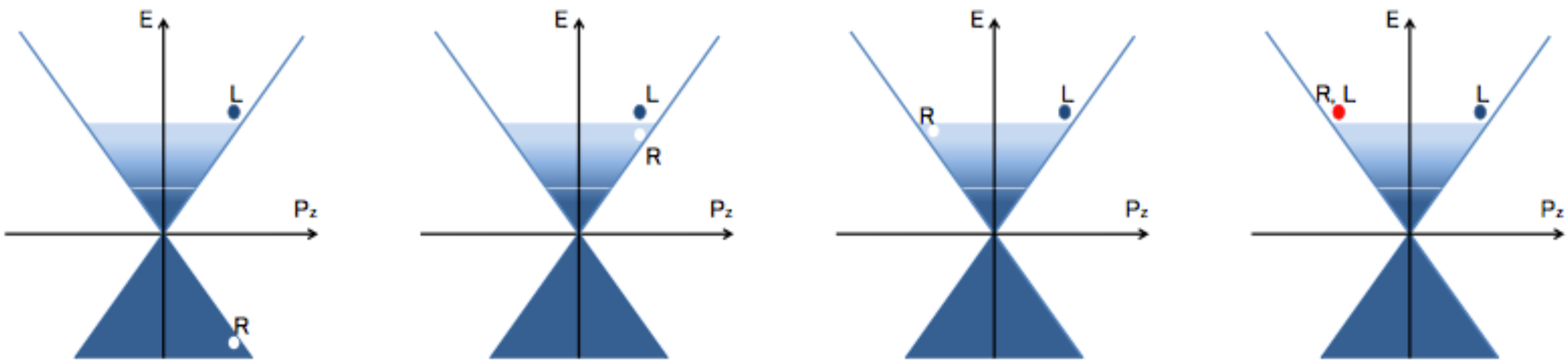

quark-antiquark

quark-hole (exciton-like)

quark-hole (CDW)

quark-quark color superconductor

Not obvious which of these is energetically favored

Unfortunately it seems that the favored condensate is somehow model dependent.
The appearance of inhomogeneous phases makes the picture even more complicated

Melting the chiral condensate

The chiral condensate becomes disfavored with increasing density

It can melt in different ways:
1) By a second order phase transition
2) By a first order phase transition
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NJL-model analysis

Pauli-Villars regulator Λ = 757.048 MeV
Coupling constant G = 6/Λ^2

variational parameters

CDW ansatz

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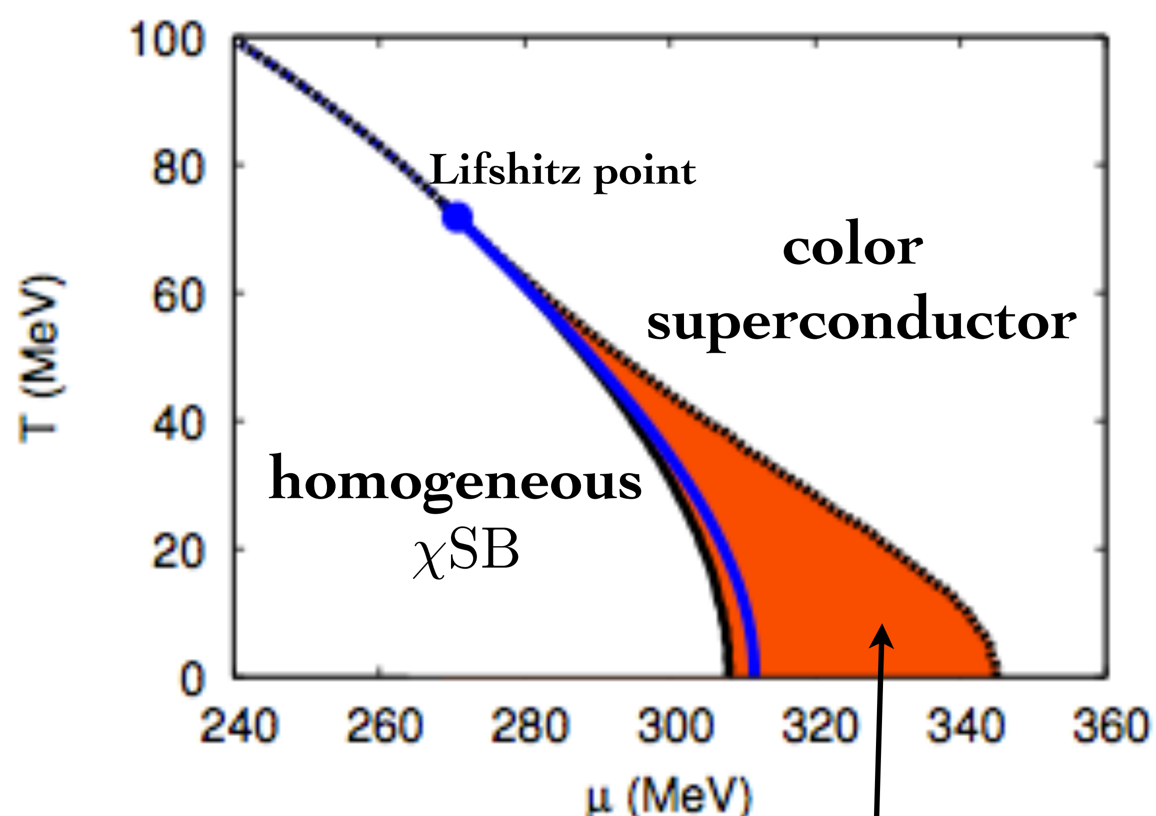

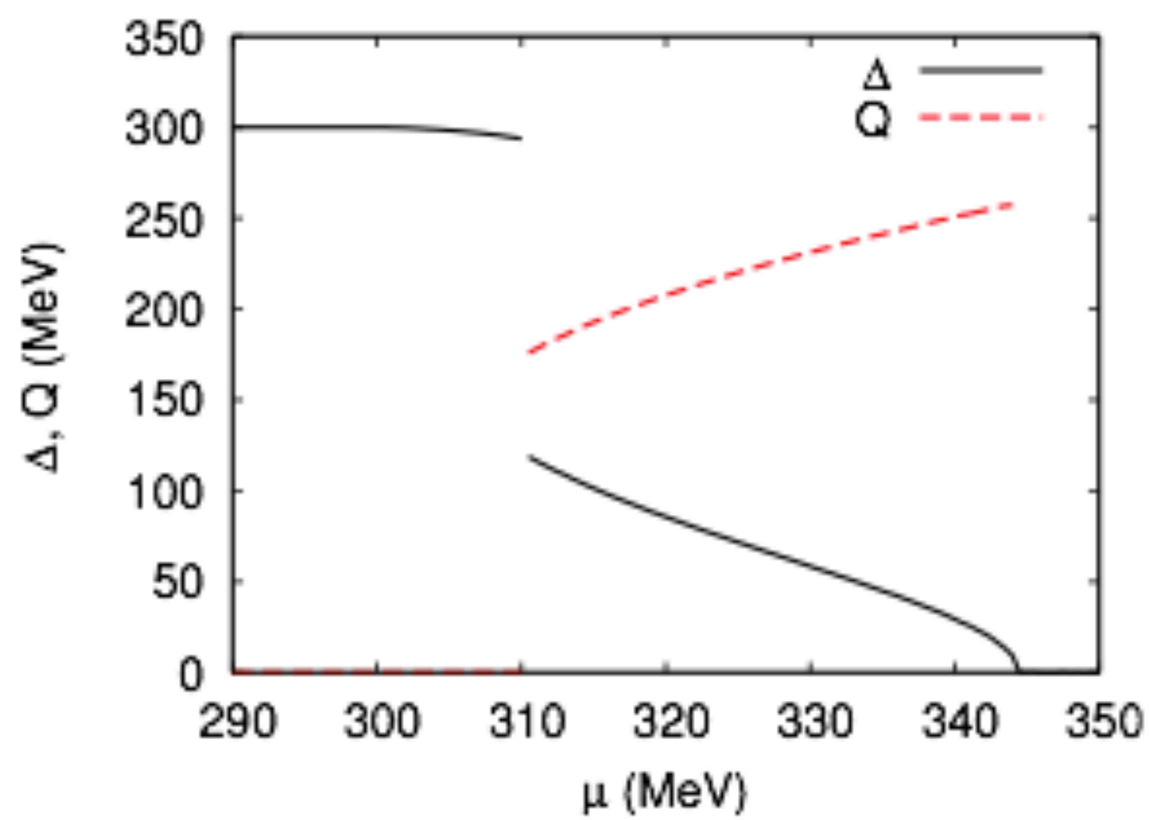

Buballa and Carignano
Prog.Part.Nucl.Phys. 81 (2015) 39-96

IMPROVED GINZBURG-LANDAU EXPANSION

Standard GL expansion

We focus on inhomogeneous chiral symmetry breaking

$$\Omega_{\rm GL} = \Omega[0] + \frac{1}{V}\int d\boldsymbol{x}\left[\alpha_2 M^2 + \alpha_4\left(M^4 + (\nabla M)^2\right) + \alpha_6\left(M^6 + 3(\nabla M)^2 M^2 + \frac{1}{2}(\nabla M^2)^2 + \frac{1}{2}(\nabla^2 M)^2\right)\right.$$
$$\left. + \alpha_8\left(14M^4(\nabla M)^2 - \frac{1}{5}(\nabla M)^4 + \frac{18}{5}M(\nabla^2 M)(\nabla M)^2 + \frac{14}{5}M^2(\nabla^2 M)^2 + \frac{1}{5}(\nabla^3 M)^2\right) + \dots\right]$$

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But is not in general true.

What is the "correct expansion" **away from the Lifshitz point**?
How to compute the relevant terms?
Which are the characteristic scales of fluctuations?

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The α_n coefficients are “universal”, for the considered system.

They shoul be derived from the microscopic theory. Since in this regime QCD is non perturbative, they depend on the effective model we use.

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Using a NJL model, they do not only depend on μ , but also on the regularization scale Λ

$$\alpha_2 = \frac{1}{4G} - \frac{N_f N_c}{8\pi^2}\left(3\Lambda^2 \log\left(\frac{4}{3}\right) - 2\mu^2\right)$$

$$\alpha_4 = -\frac{N_f N_c}{16\pi^2}\log\left(\frac{32\mu^2}{3\Lambda^2}\right)$$

$$\alpha_6 = \frac{N_f N_c}{96\pi^2}\left(\frac{11}{3\Lambda^2} + \frac{1}{\mu^2}\right)$$

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Even within the NJL model they are not easy to compute. **Brute force is not very rewarding**.

Improved GL expansion (for chiral symmetry breaking)

Idea: scale separation between slow and fast fluctuations + use a simple model to compute some of the coefficients.

Long wavelengths: dominant at the onset of the inhomogeneous phase
Short wavelengths: dominant at the transition to the normal phase.

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Captures short-wavelength oscillations by larger number of gradients.

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Computing the additional terms

We compute the $\tilde{\alpha}_{2n+2}$ using a simple model

We must have an analytical or semi-analytical expression of the free energy in one simple case:

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Double expansion

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In powers of q/μ

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$$\left. + \left(\frac{1}{10} - \frac{17\mu^4}{27\Lambda^4}\right)\left(\frac{q}{\mu}\right)^6 + \left(\frac{1}{21} + \frac{230\mu^6}{567\Lambda^6}\right)\left(\frac{q}{\mu}\right)^8 + \ldots \right]$$

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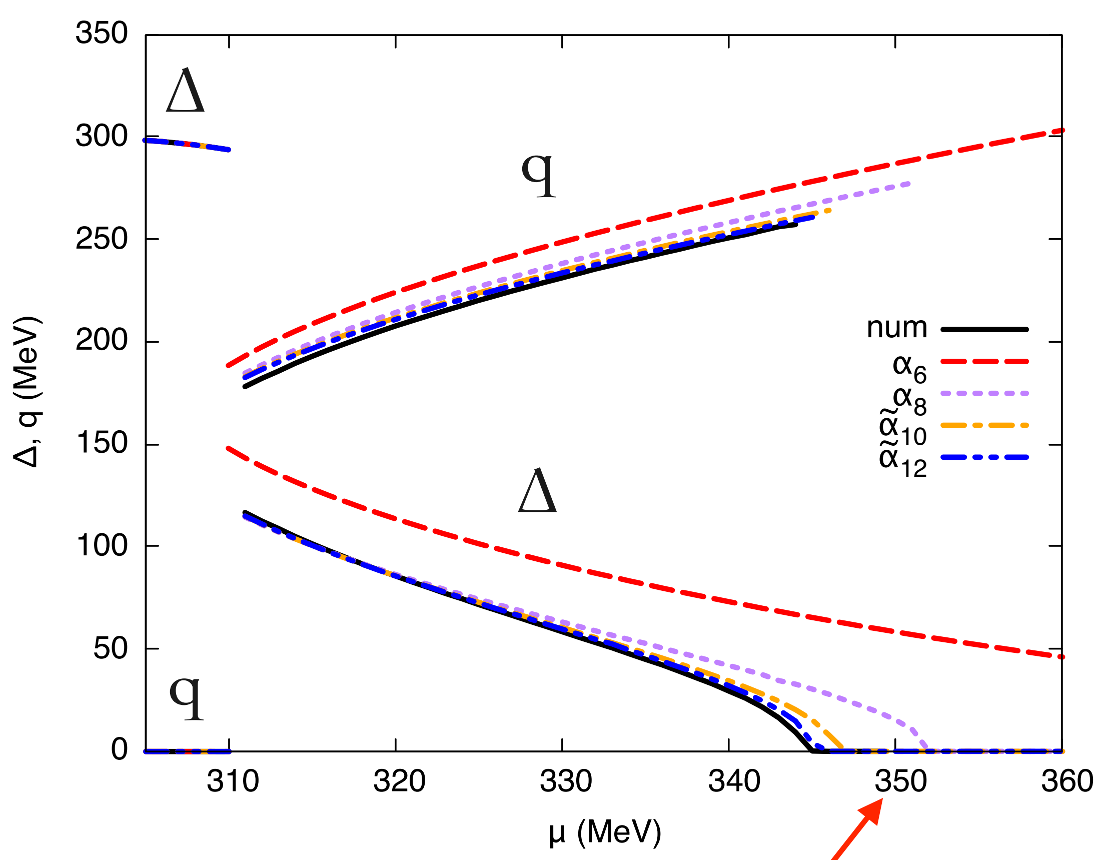
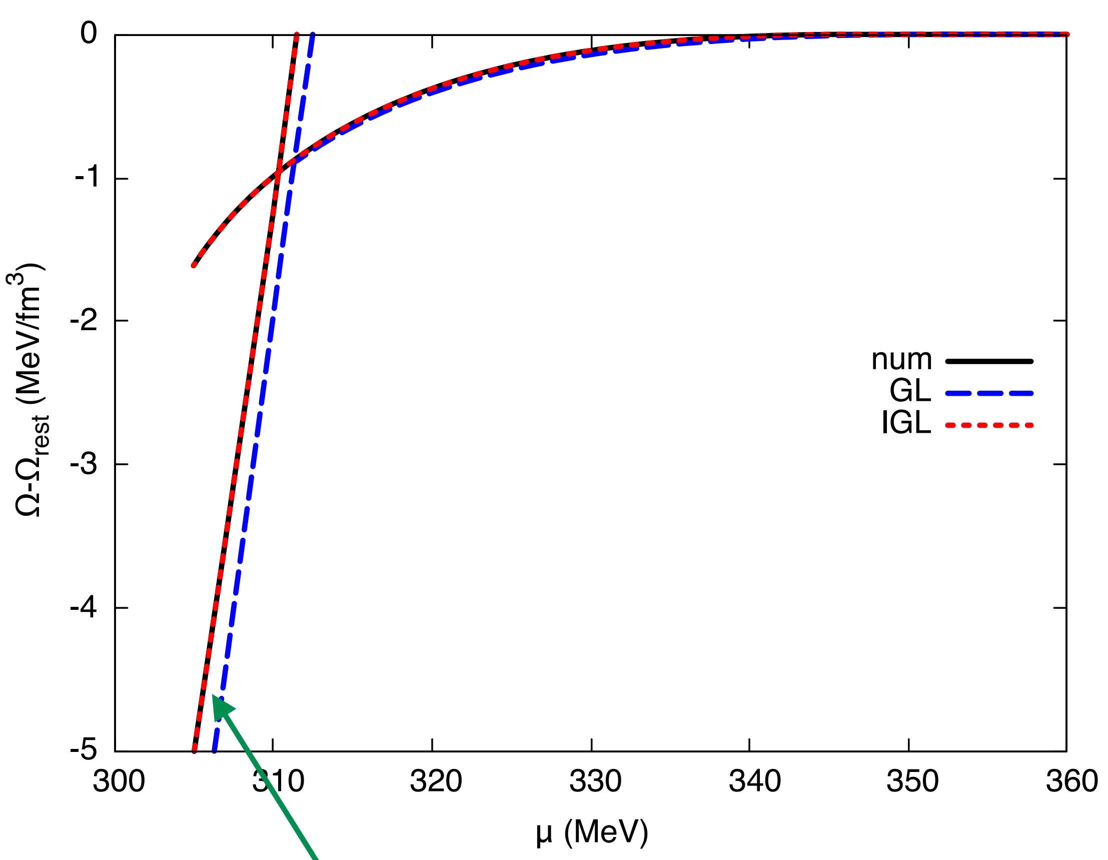
Therefore $\quad \tilde{\alpha}_{10} = \frac{N_f N_c}{1024\pi^2}\left(\frac{230}{567\Lambda^6} + \frac{1}{21\mu^6}\right) \quad$ and we can in principle extract more terms

Comparison: CDW case

Let us see what happens for the CDW anstaz $M(z) = \Delta e^{2iqz}$
In this case we have the numerical solution.





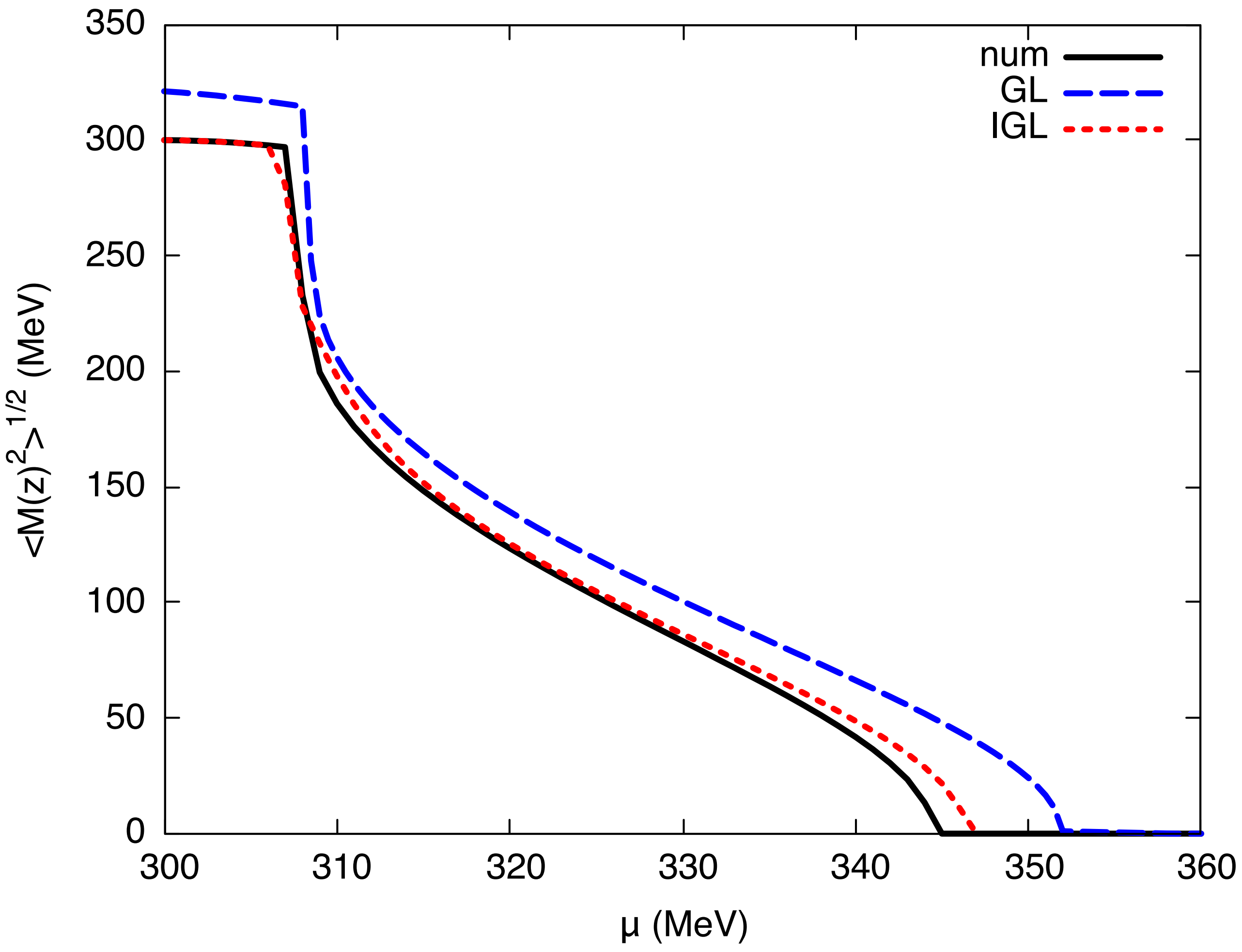
Improvement due to the inclusion of higher order terms

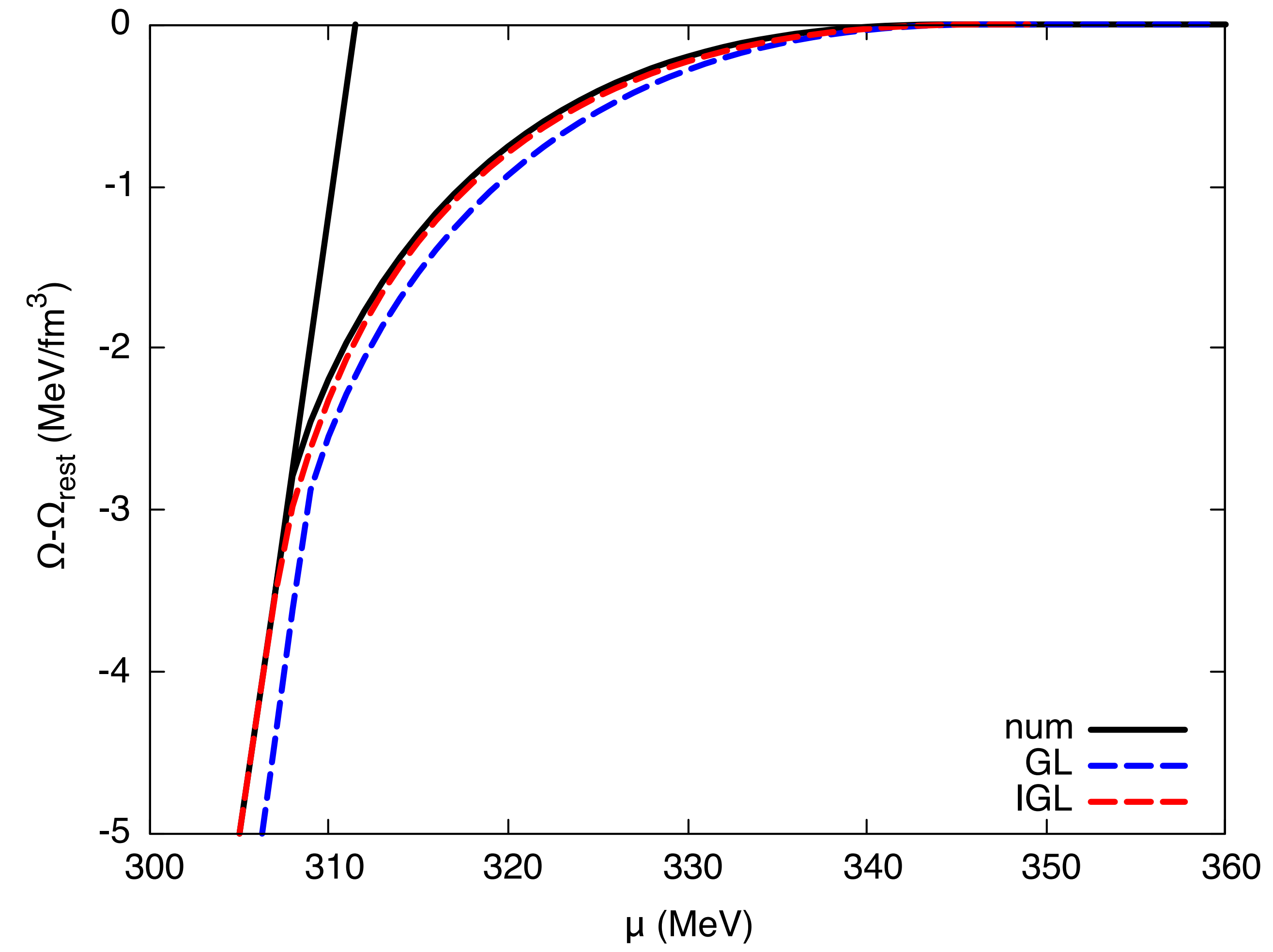
By construction IGL reproduces the homogeneous phase

Comparison: kink case

Real kink

$$M(z) = \Delta\sqrt{\nu}\,\mathrm{sn}(\Delta z|\nu)$$





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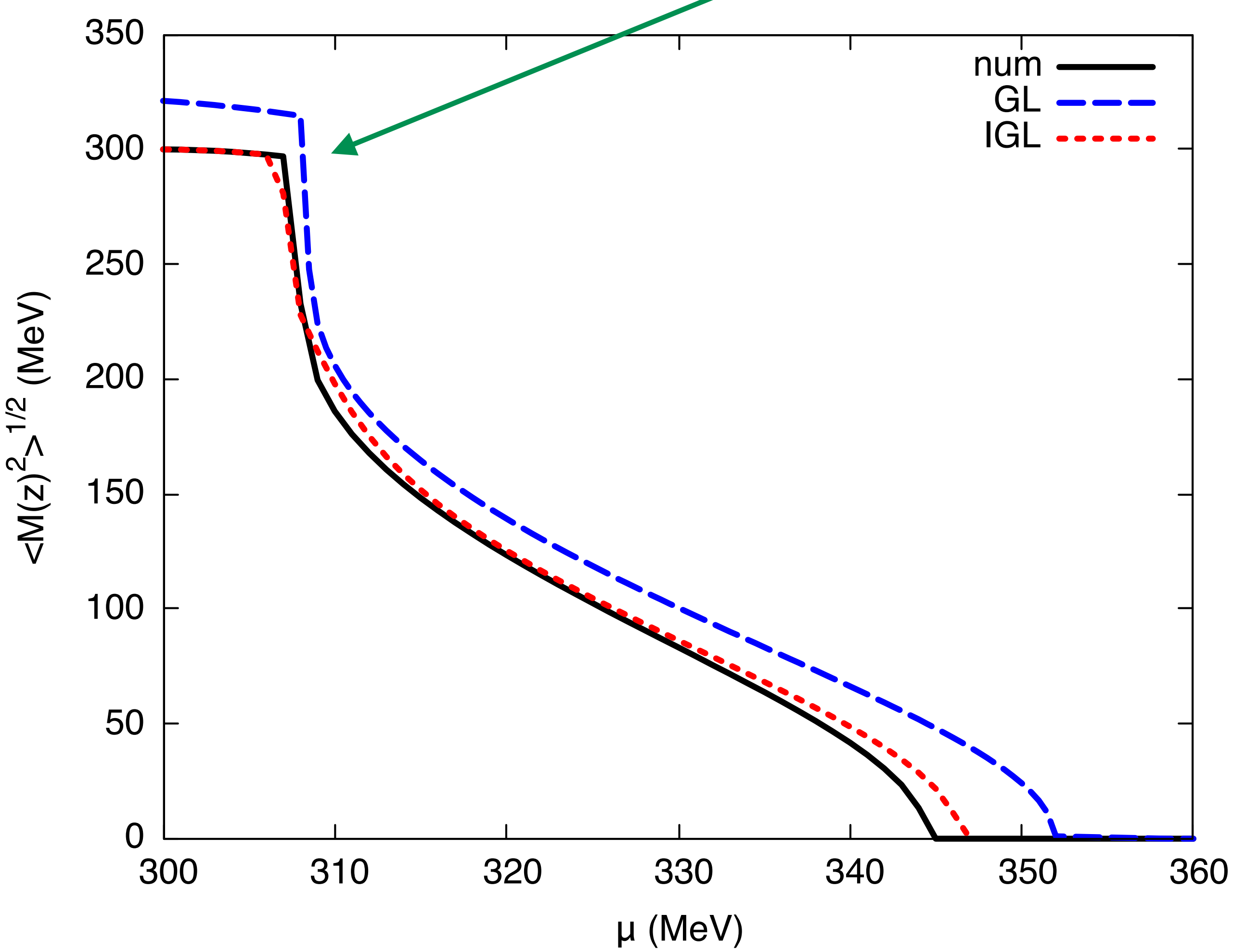
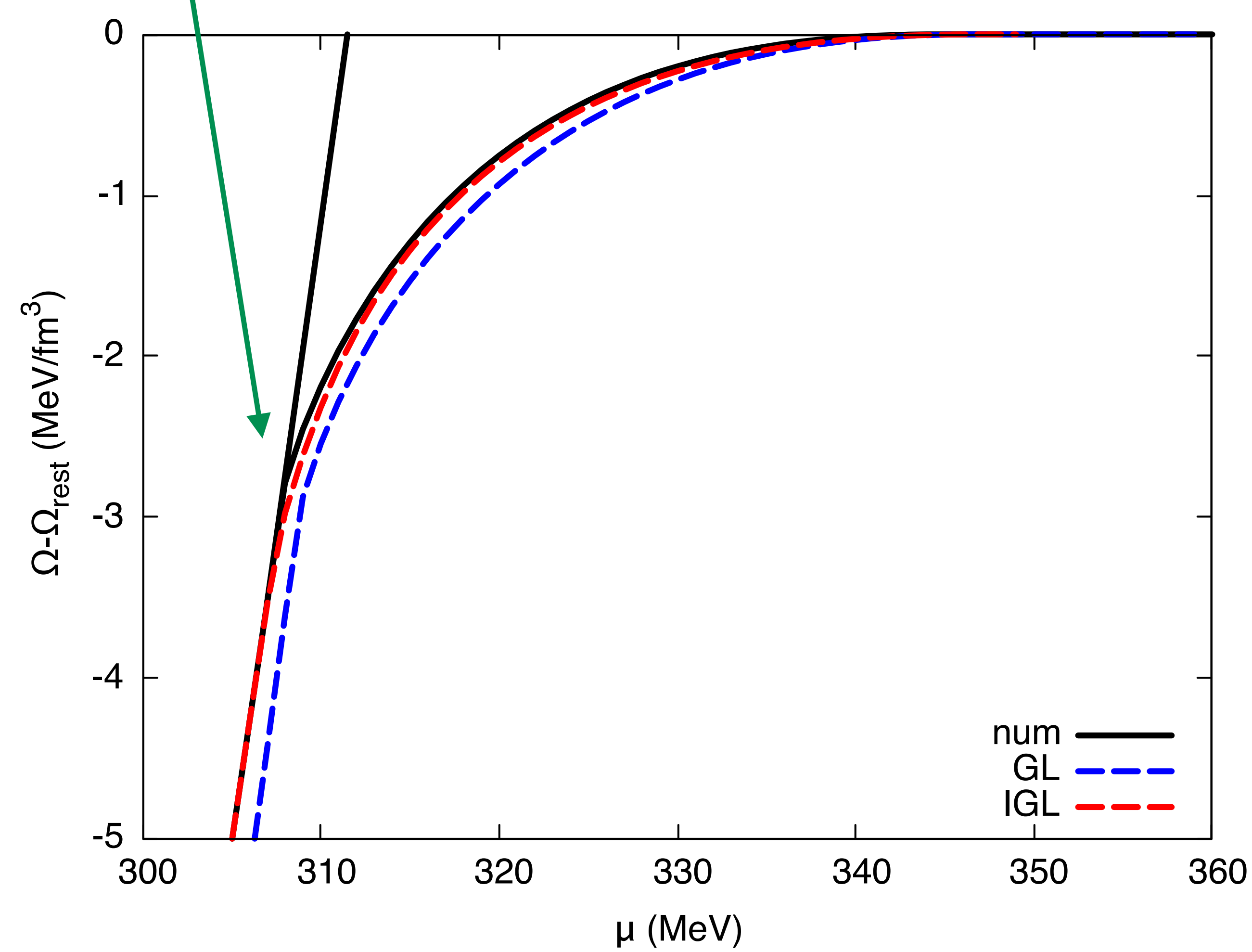
The IGL smoothly leads to the homogeneous phase





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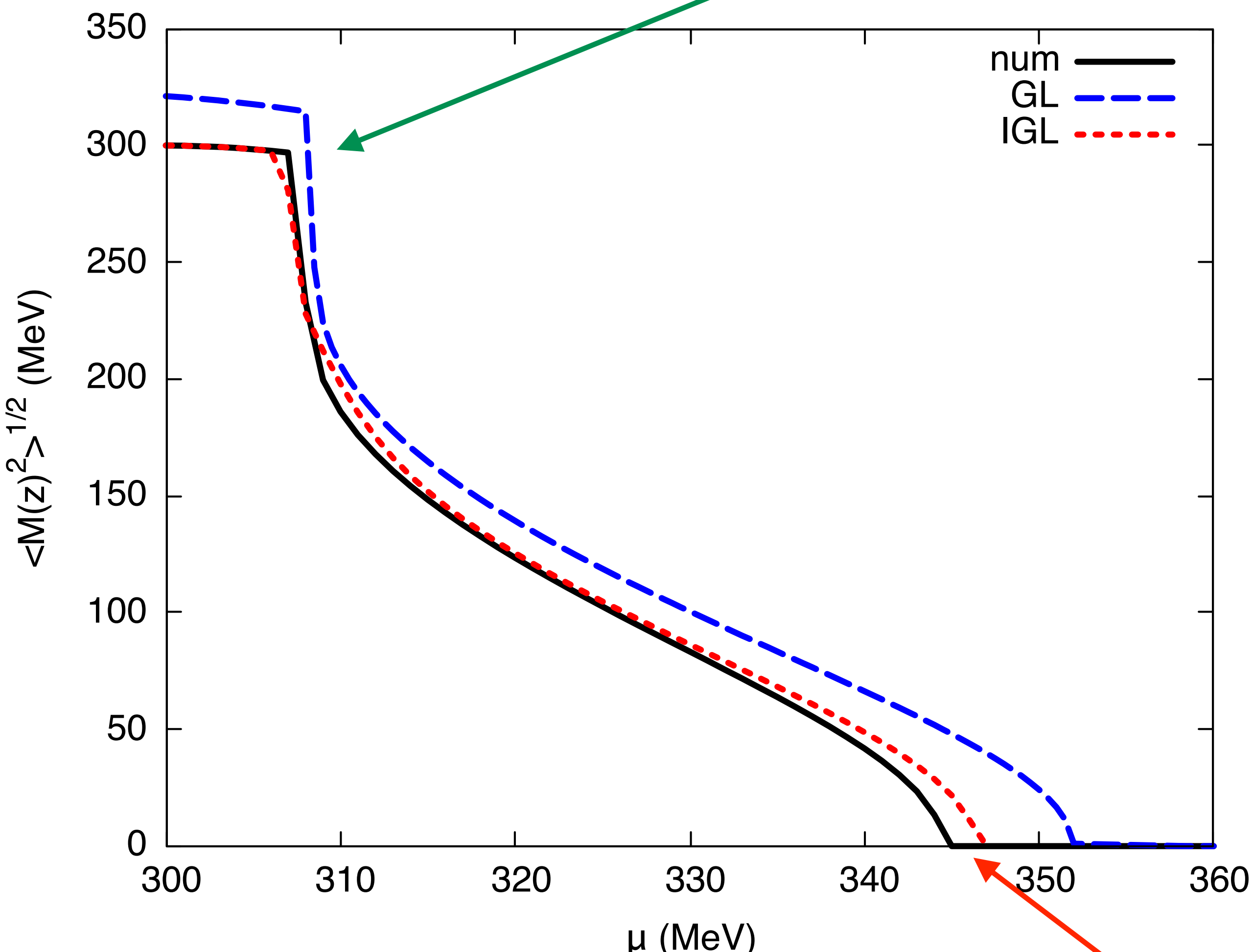
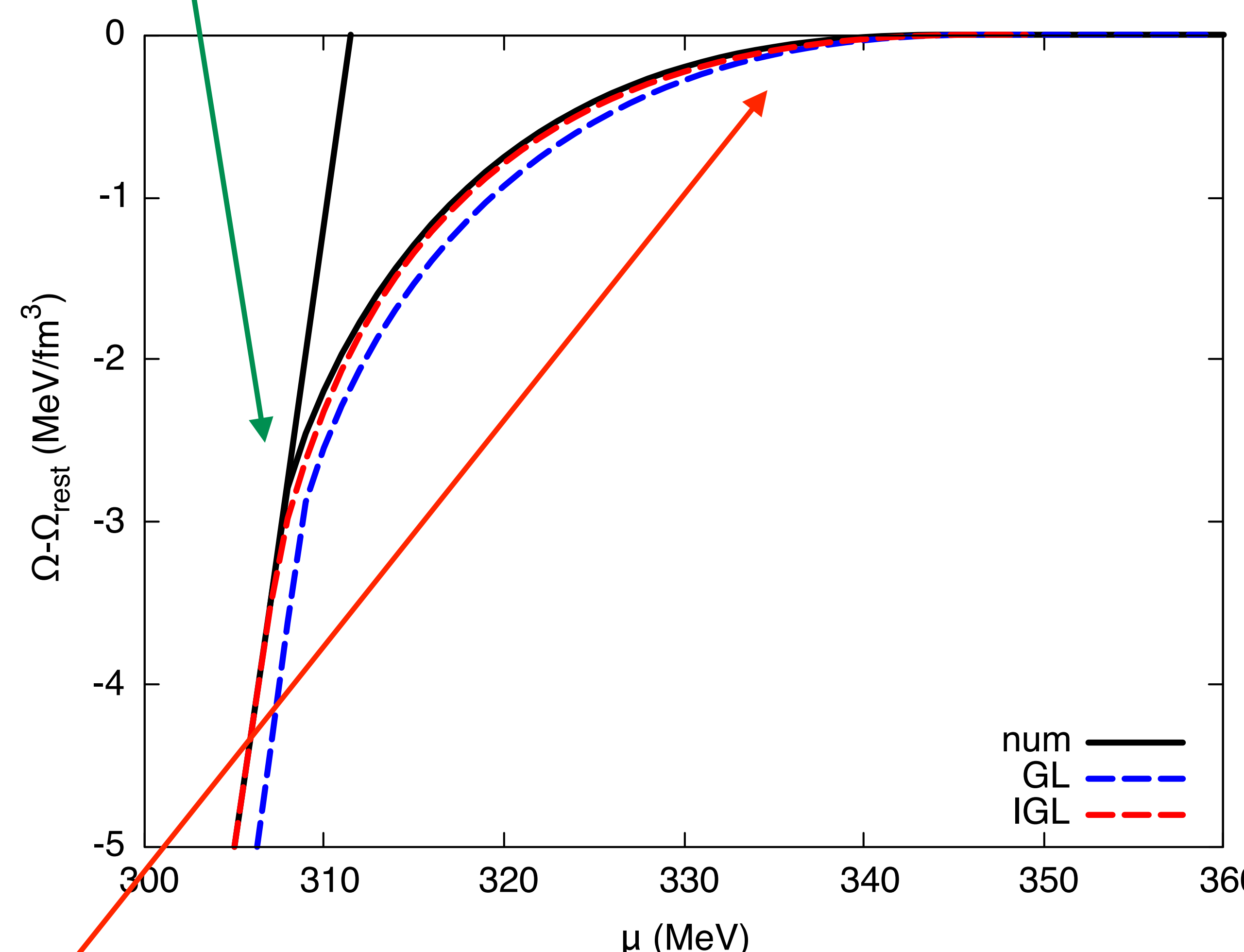
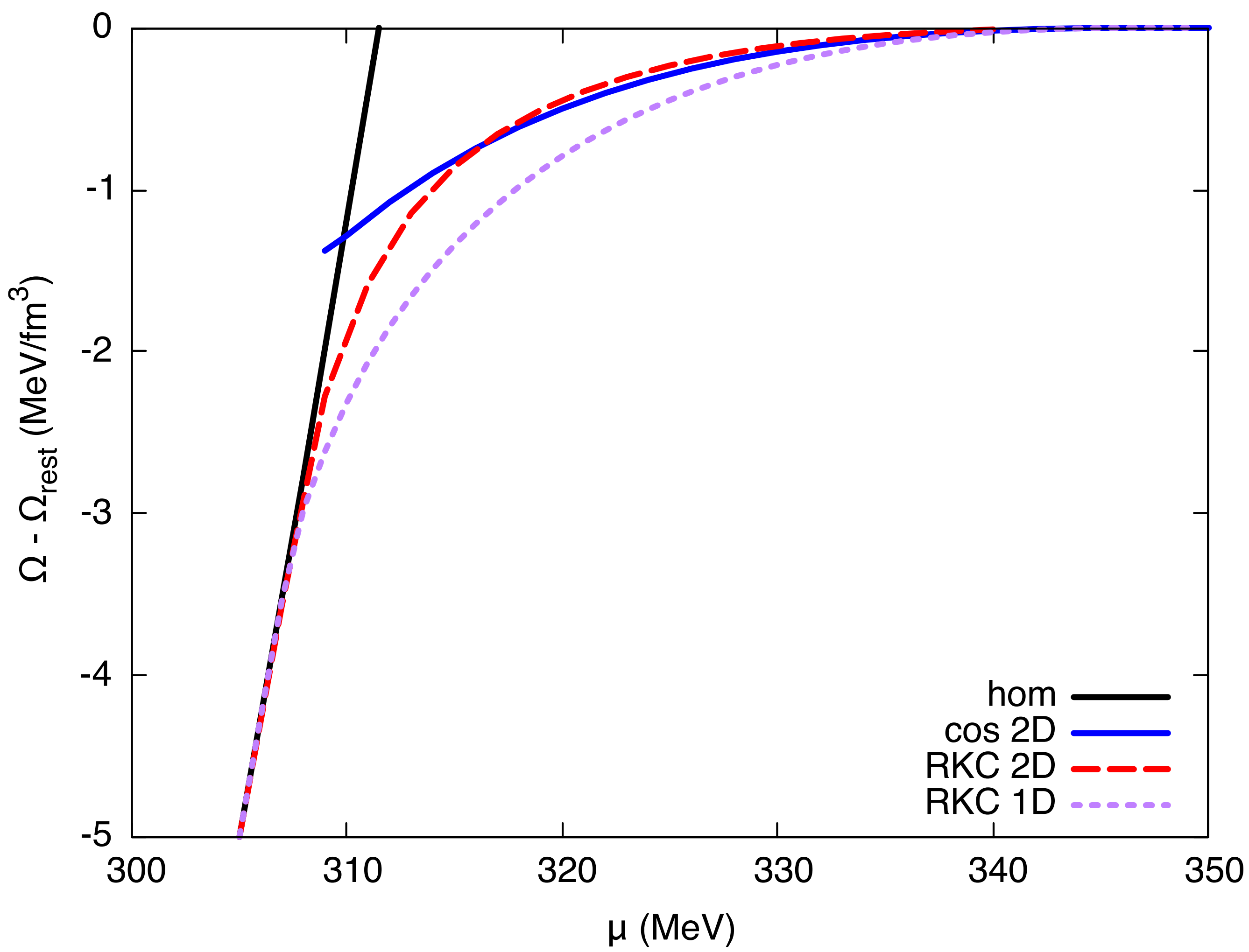




Improvement due to the inclusion of higher order terms

Comparison of some 1D and 2D modulations

Free energy of various phases in the IGL approximation



Why 1D modulations always win? Where does pairing occur?

Qualitative analysis of pairing

We closely inspect the integrand of the CDW ansatz

$$\Omega_{\rm CDW} = -\frac{N_f N_c}{4\pi^2}\int_0^\infty dp_\perp p_\perp \int_{-\infty}^{\infty} dp_z \sum_{\epsilon=\pm}\Big[E^\epsilon_{PV} + (\mu - E^\epsilon)\theta(\mu - E^\epsilon)\Big] + \frac{\Delta^2}{4G}$$

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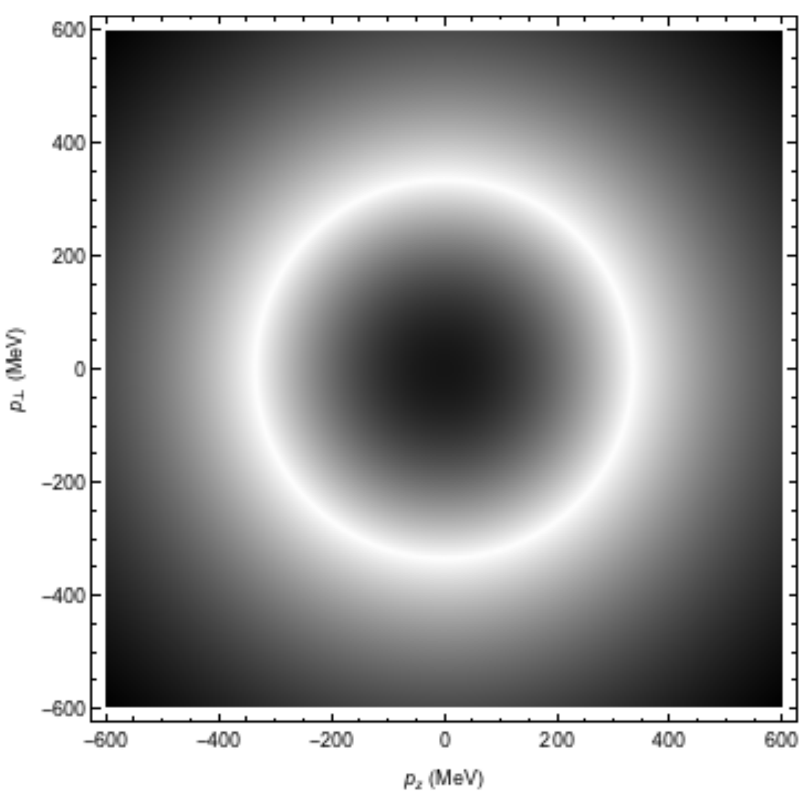
2D projection of the Fermi spheres for $\mu = 335$ MeV.
Light region: the energy cost for exciting quasiparticle is small

$\Delta = 0, Q = 0$



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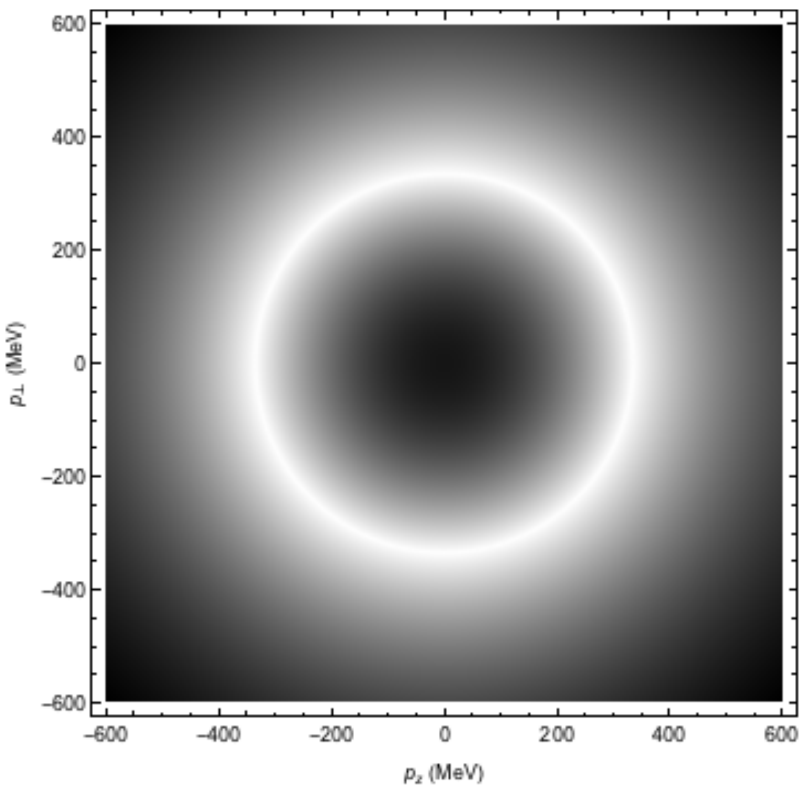

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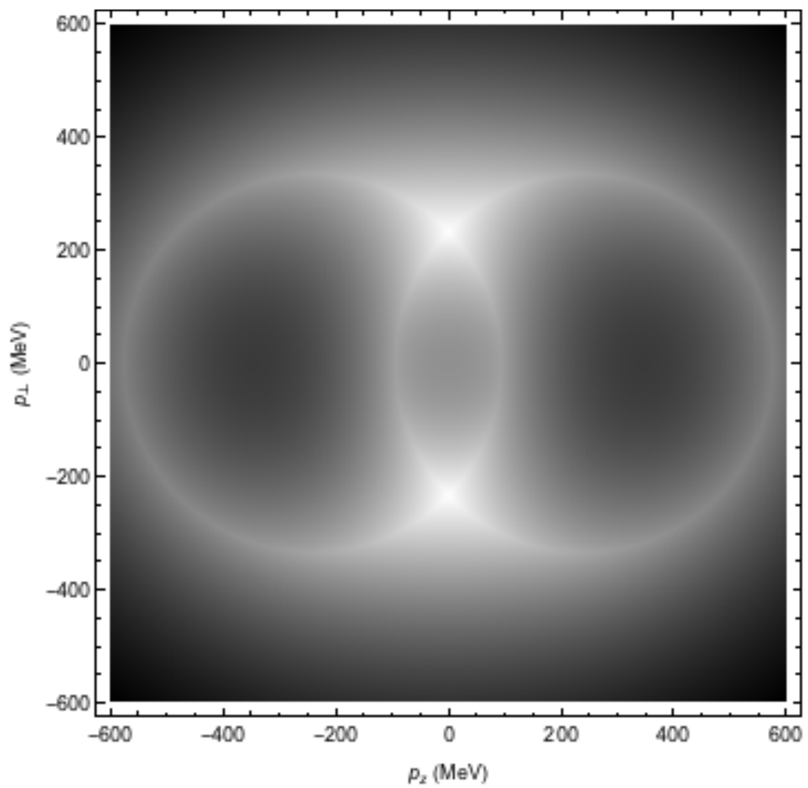

2 displaced Fermi spheres

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Light region: the energy cost for exciting quasiparticle is small

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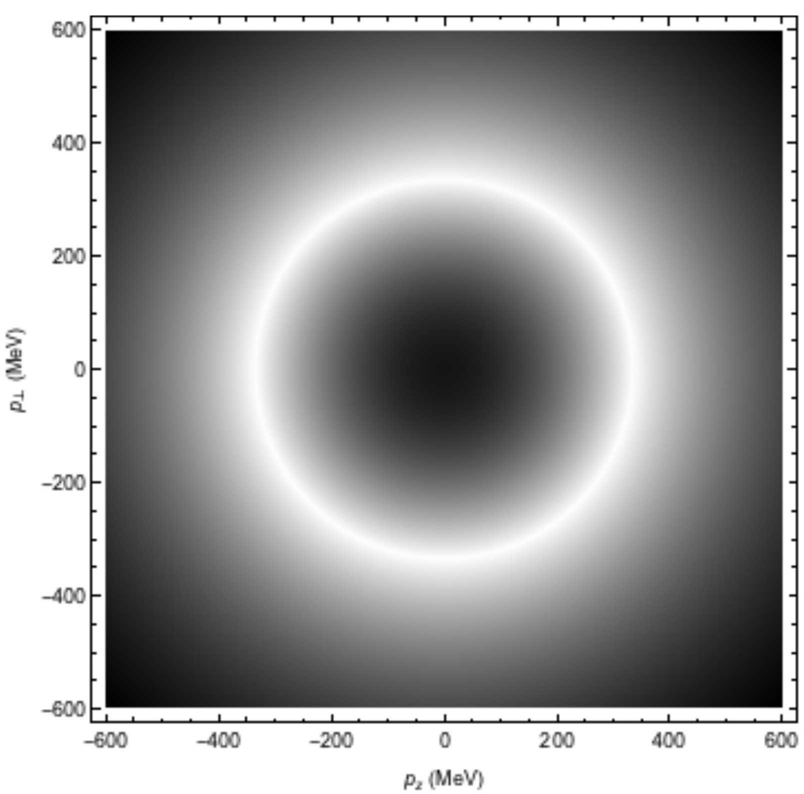
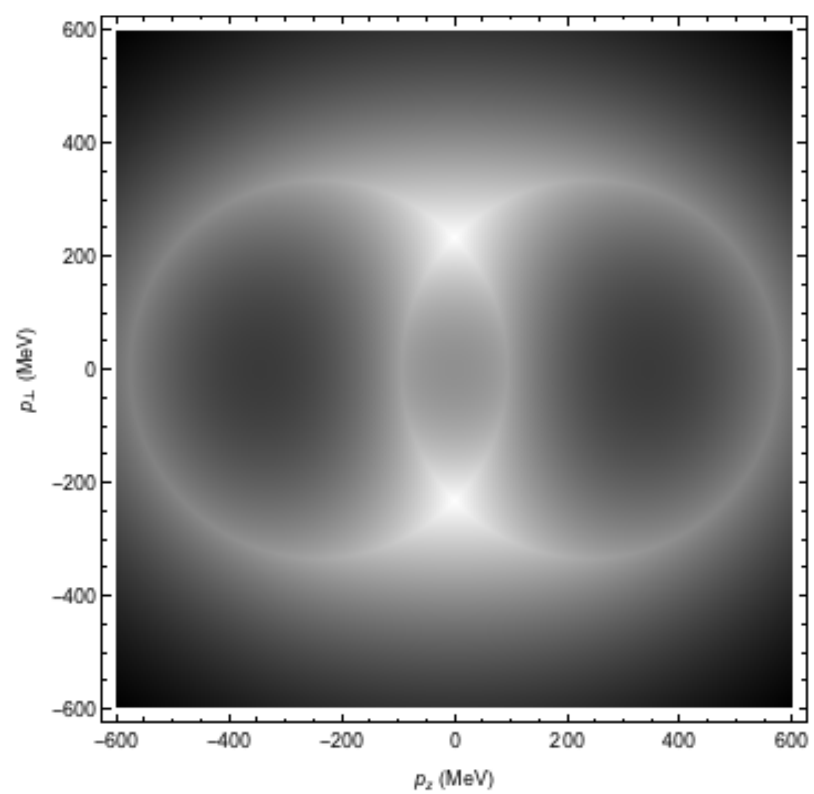


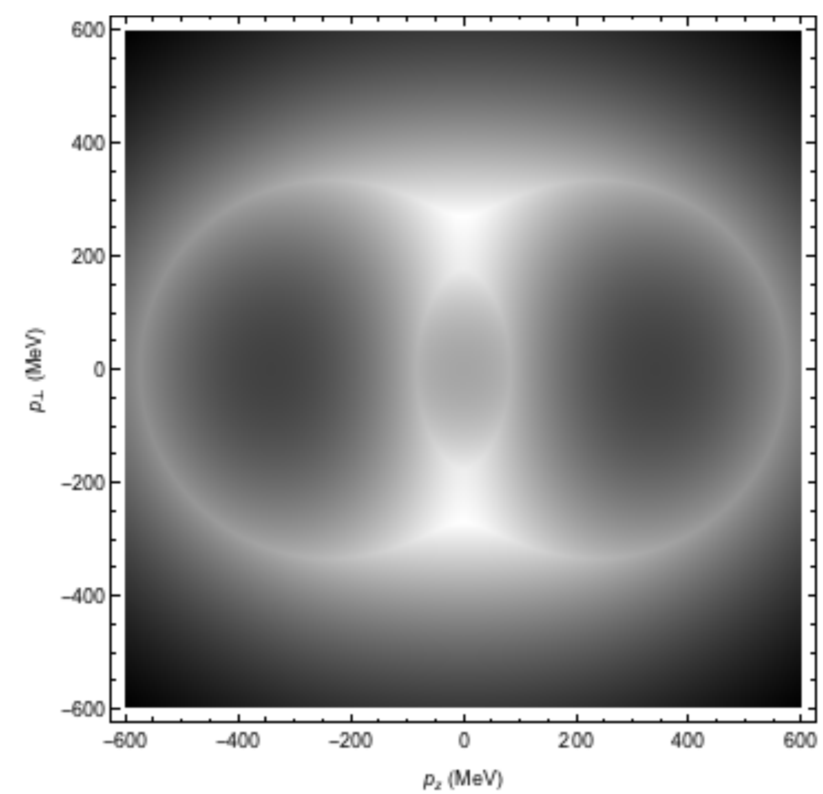
2 coincident Fermi spheres

$\Delta = 0, Q = 241$ MeV



2 displaced Fermi spheres

$\Delta = 44$ MeV$, Q = 241$ MeV

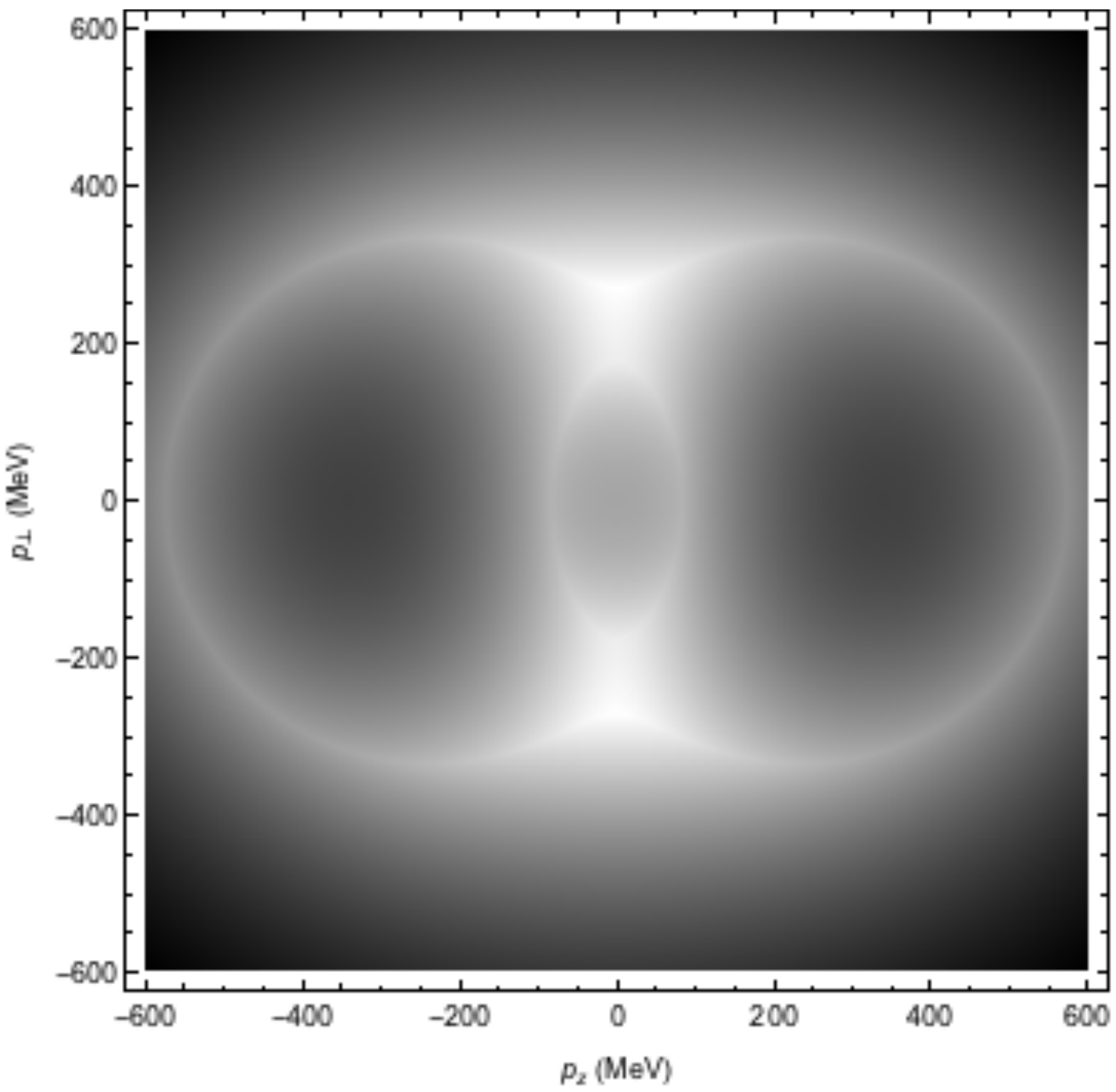


Turning on pairing

Analysis of pairing

The Fermi spheres are strongly modified. As far as Q is large, only 1D modulation can be favored

$$\Delta = 44 \text{ MeV}, Q = 241 \text{ MeV}$$



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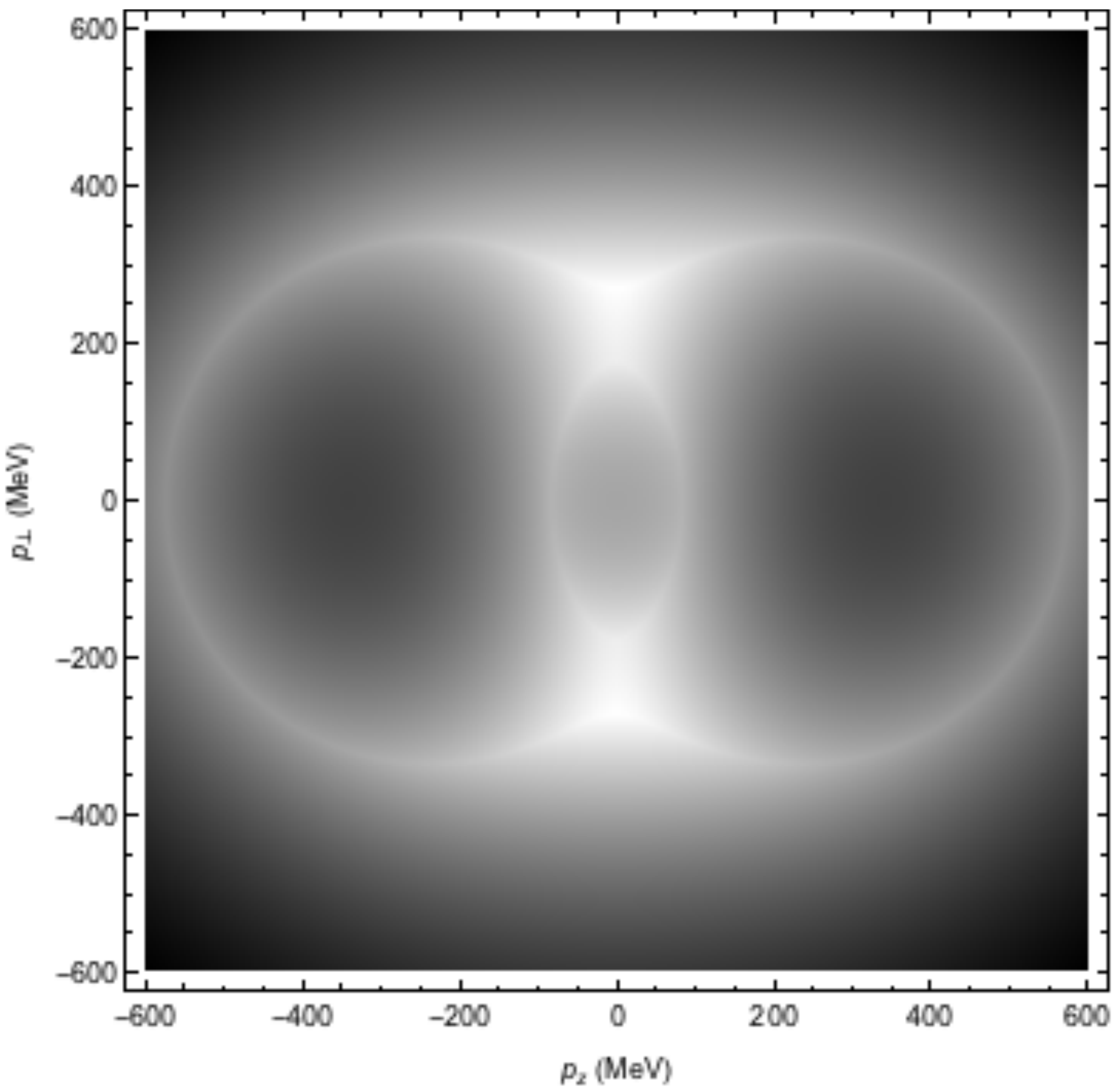

For example: pairing

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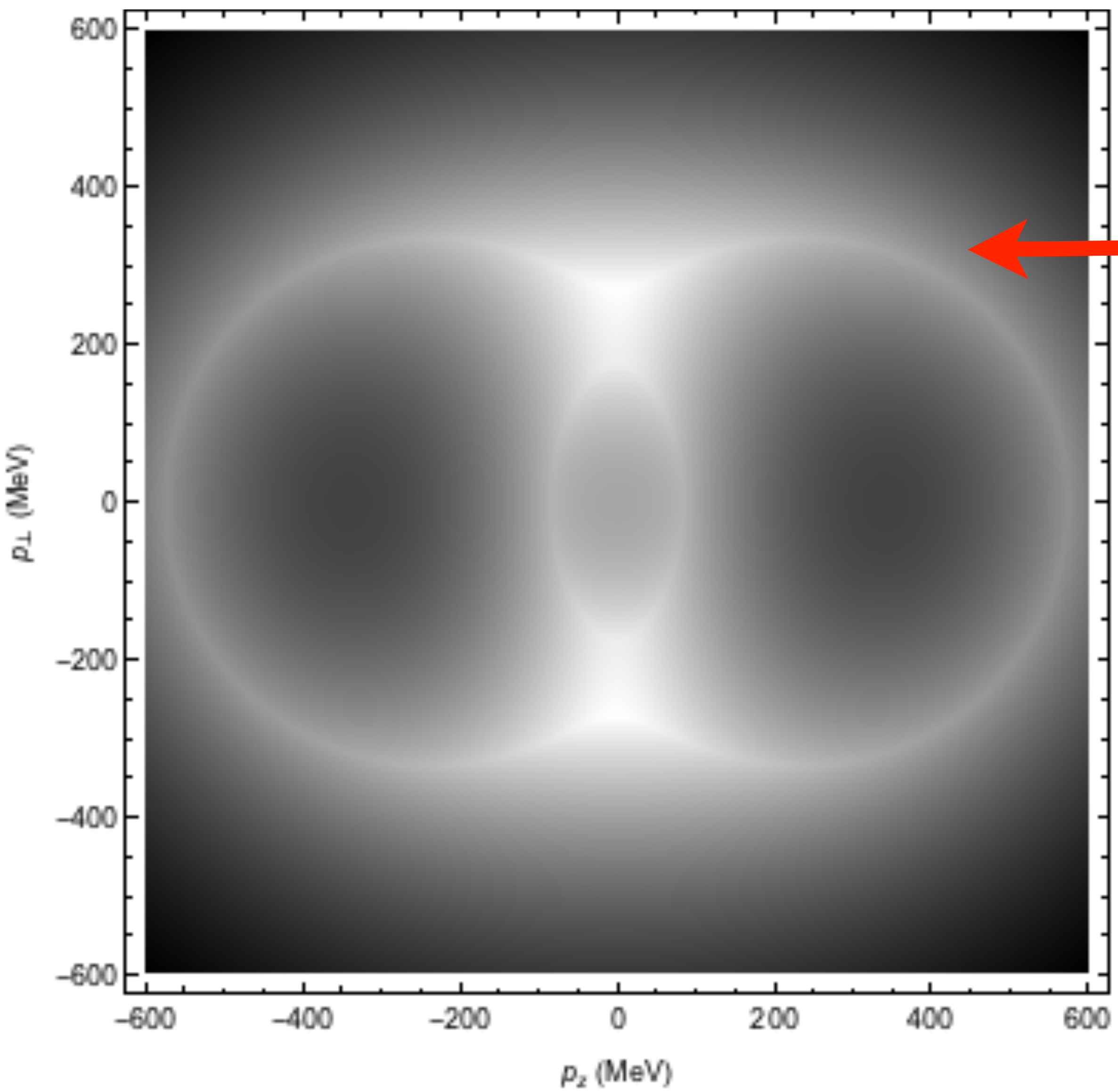

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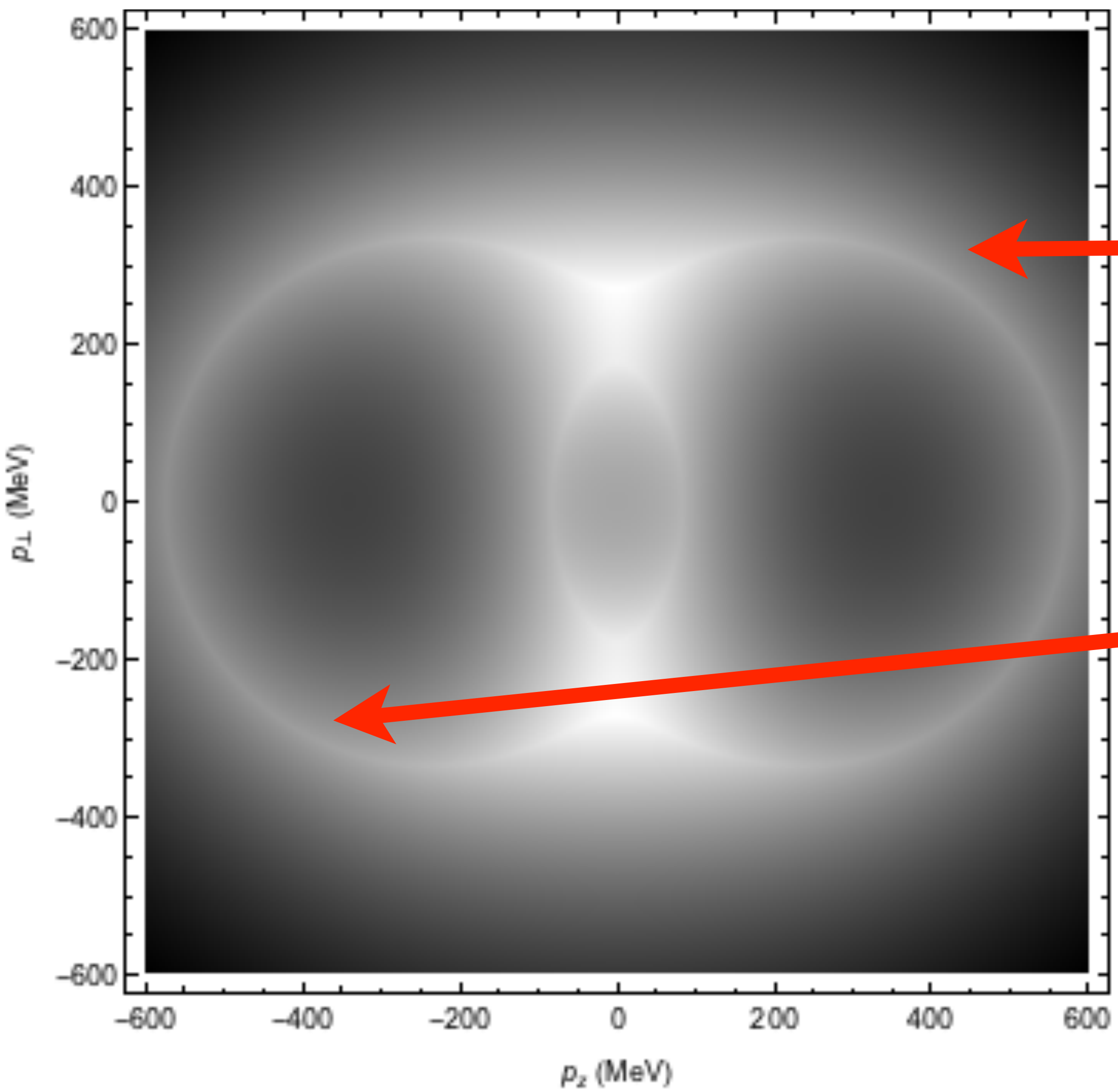

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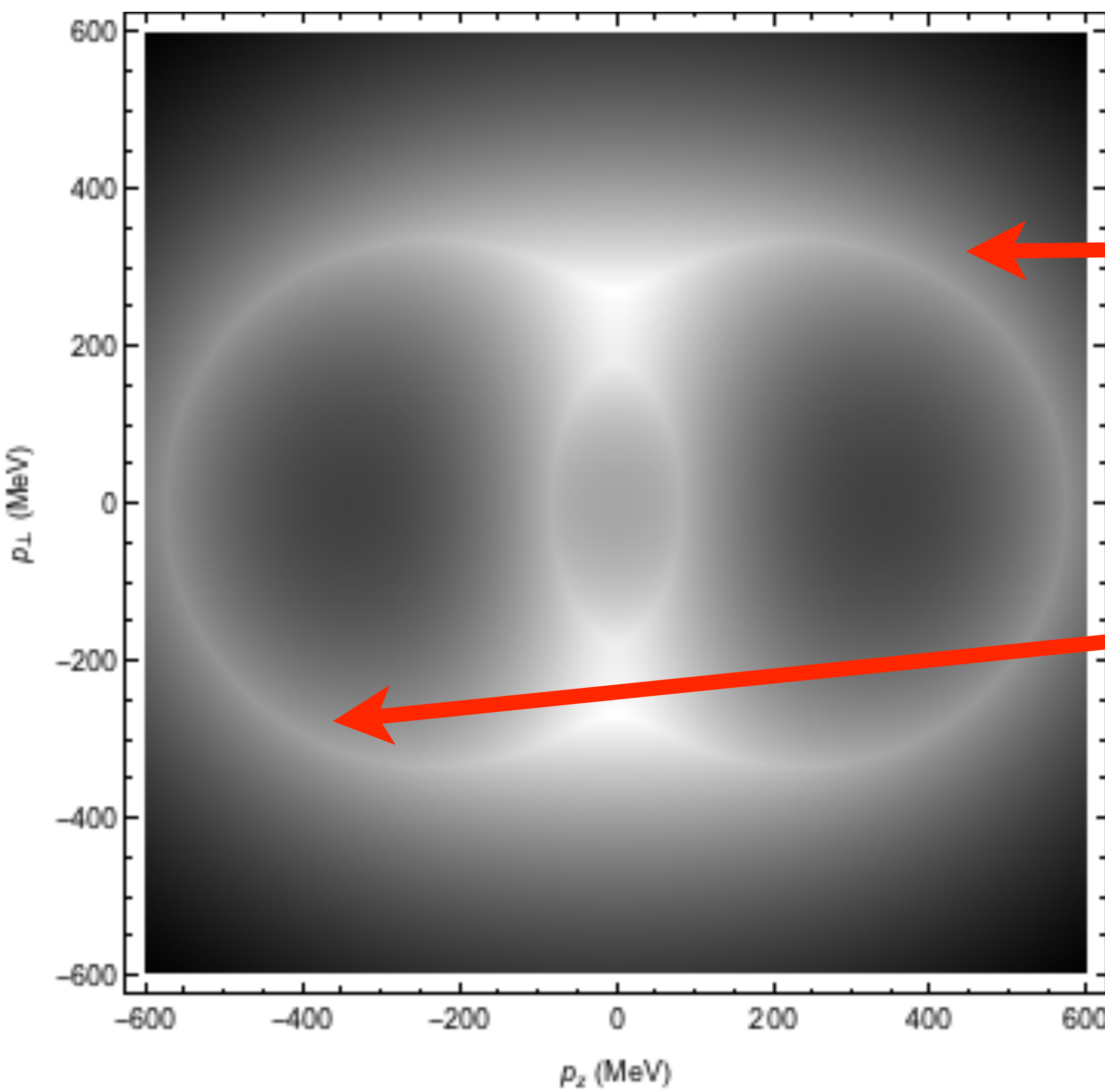

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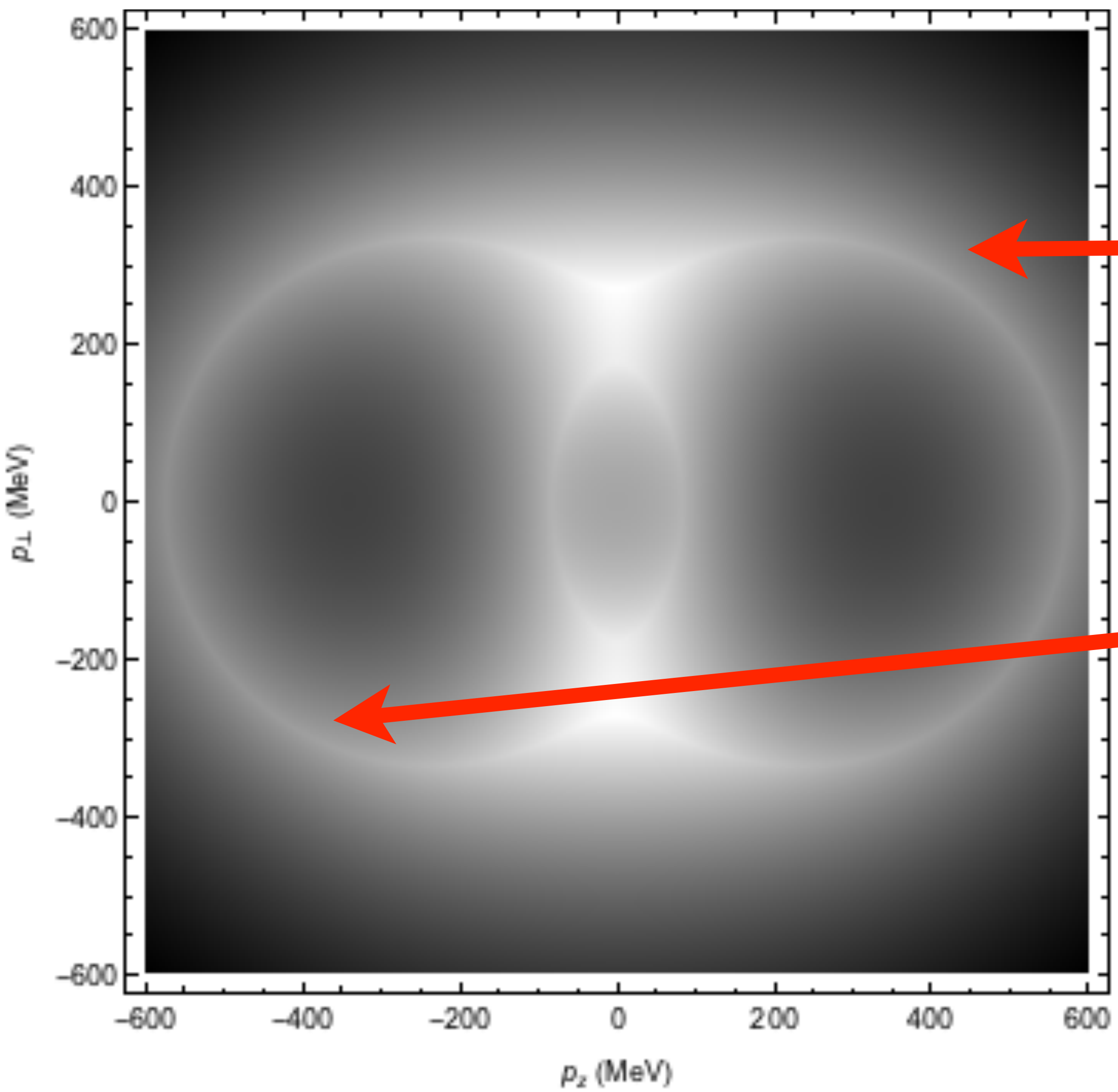

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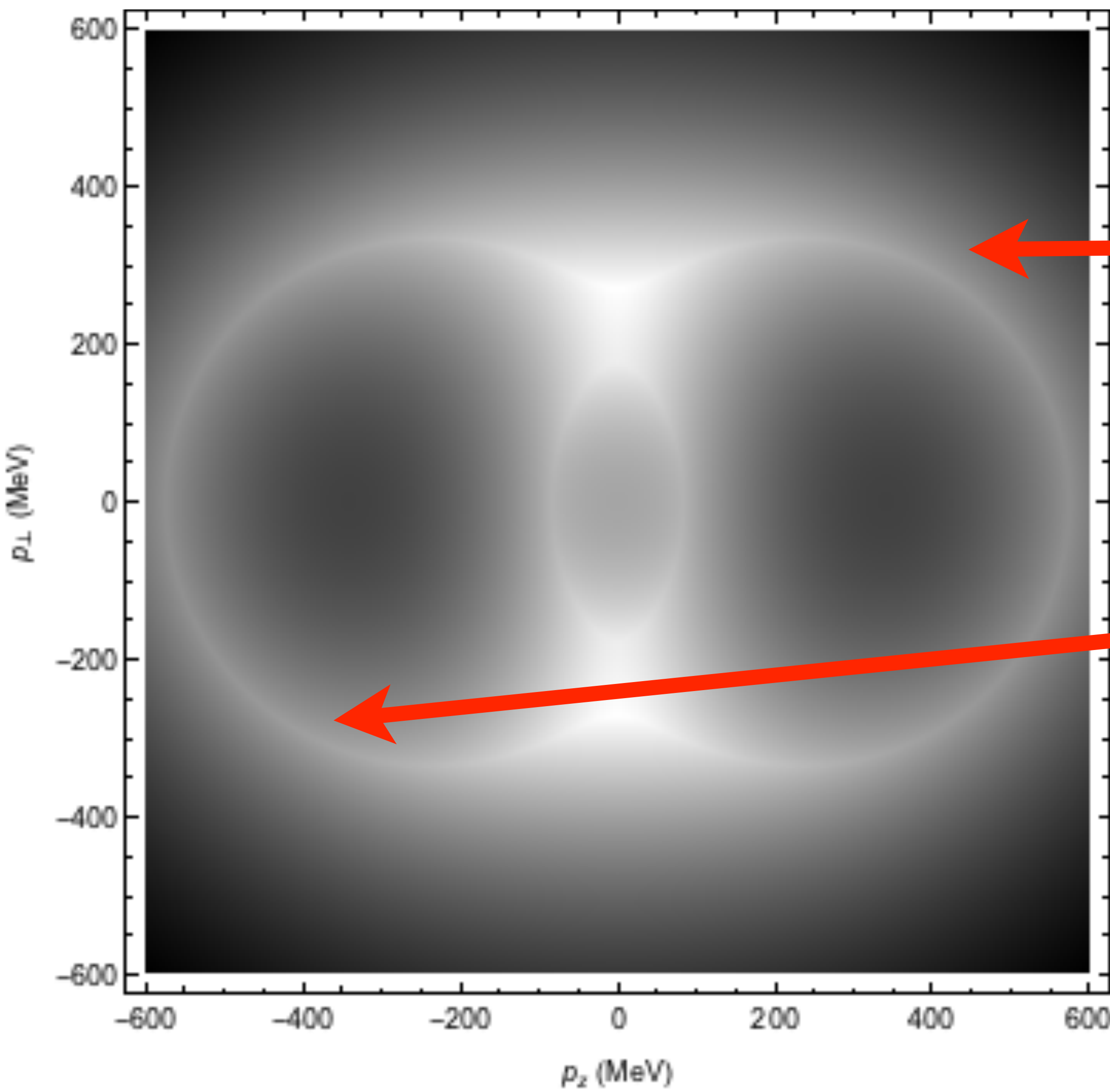

For example: pairing

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with

quarks here

requires a large deformation of the Fermi sphere

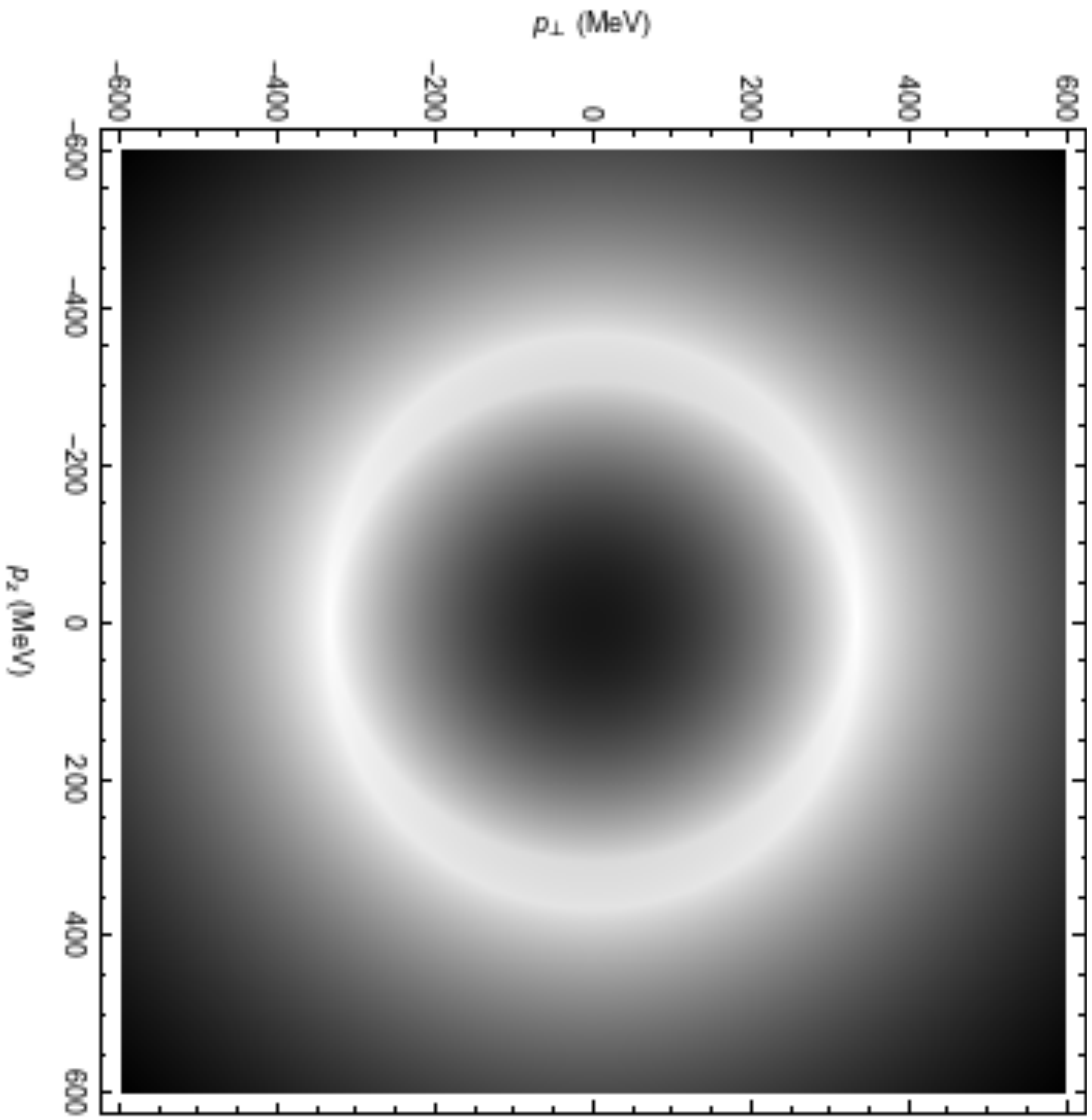
There seems no way to have a crystalline phase.

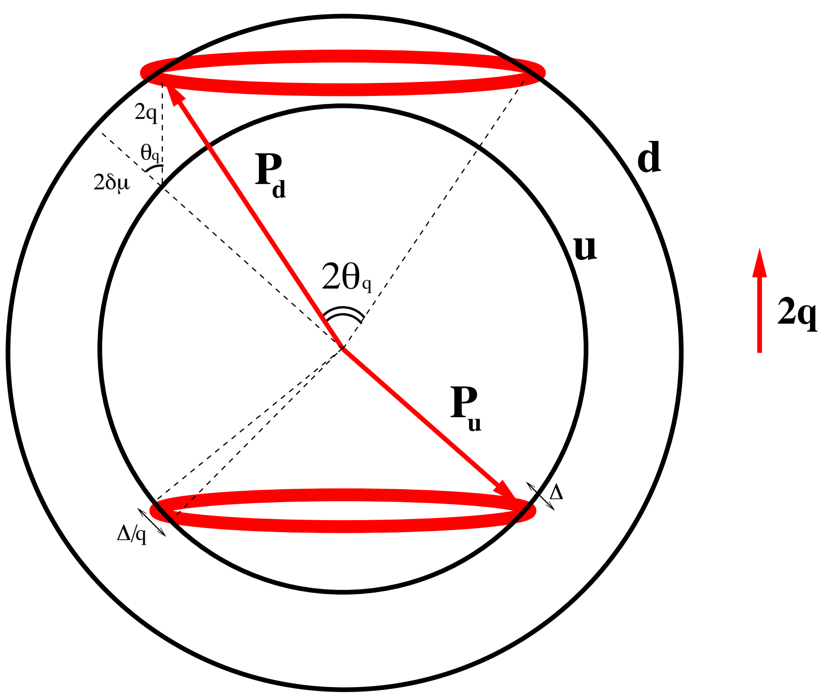
But then why a crystalline phase is realized in color superconductors?

FFLO-phase (quark-quark pairing)

Mismatched two flavor quark matter

For $\delta\mu_1 < \delta\mu < \delta\mu_2$ **the superconducting phase with Cooper pairs of non-zero total momentum is favored**





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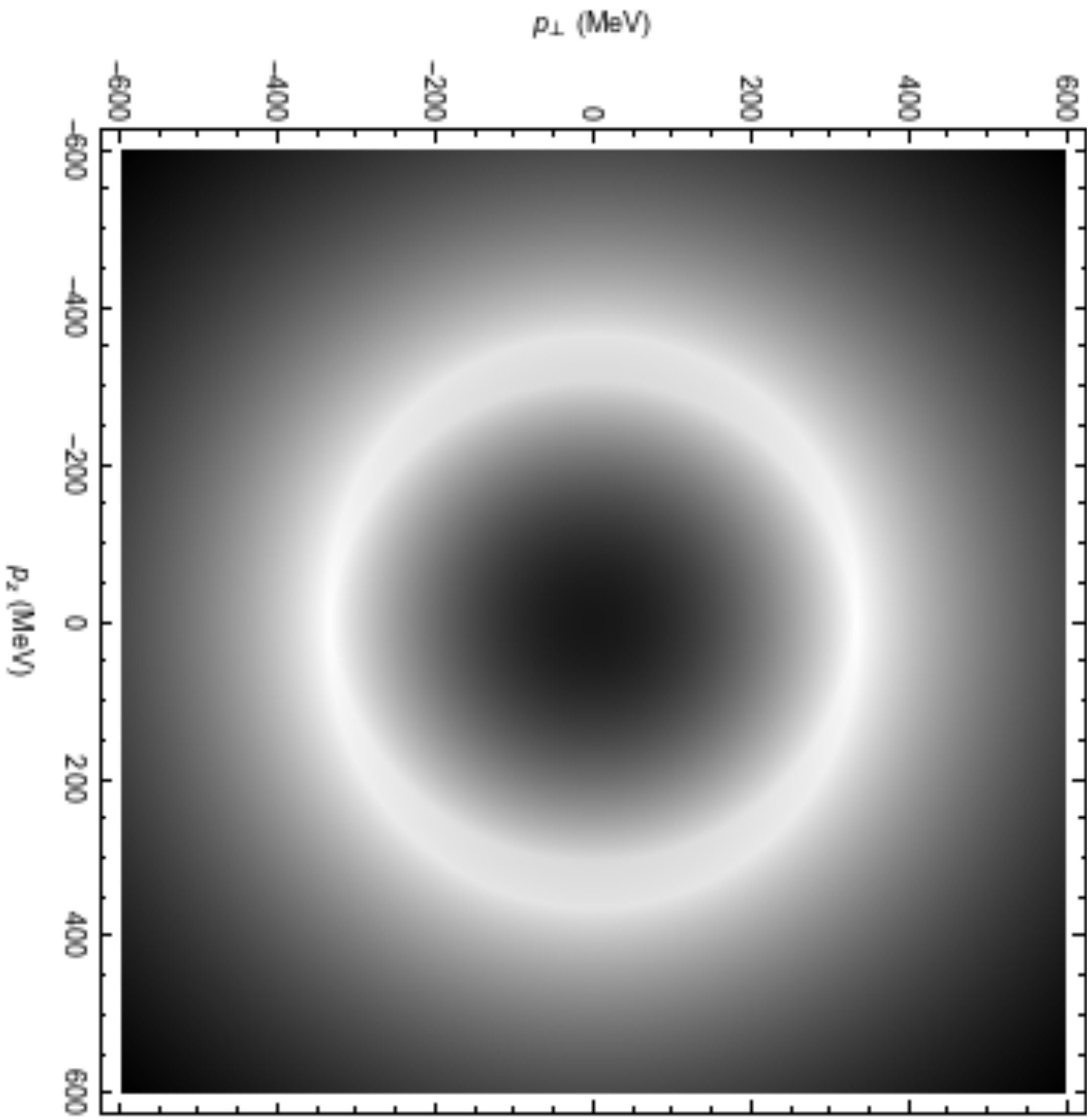

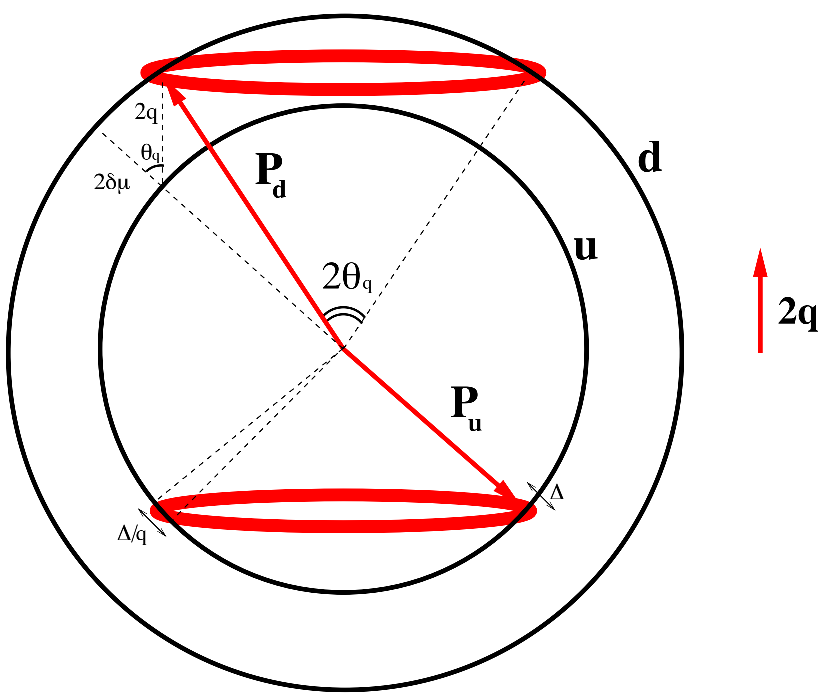

In weak coupling

$$\delta\mu_1 \simeq \frac{\Delta_0}{\sqrt{2}} \qquad \delta\mu_2 \simeq 0.75\,\Delta_0$$

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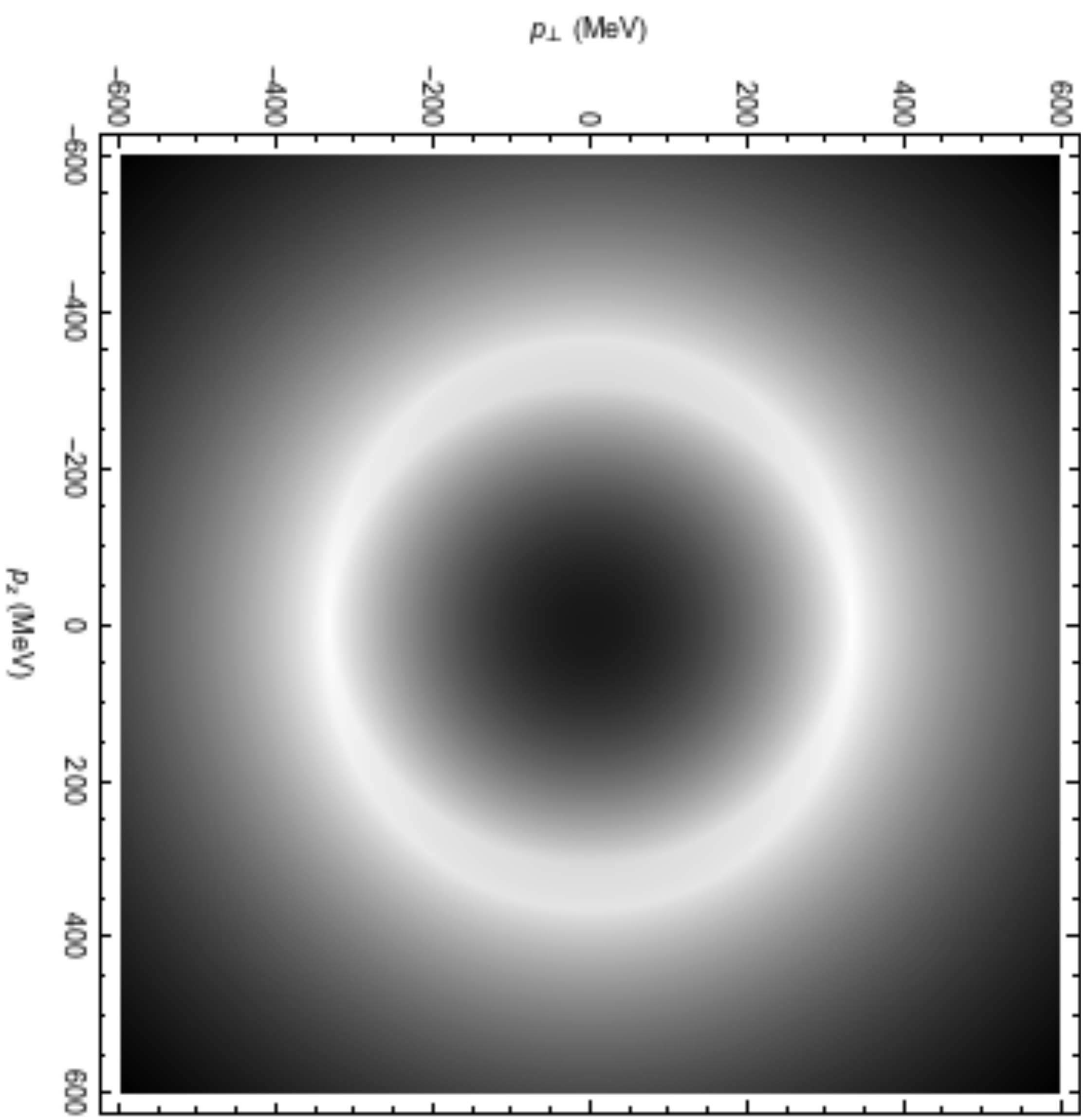

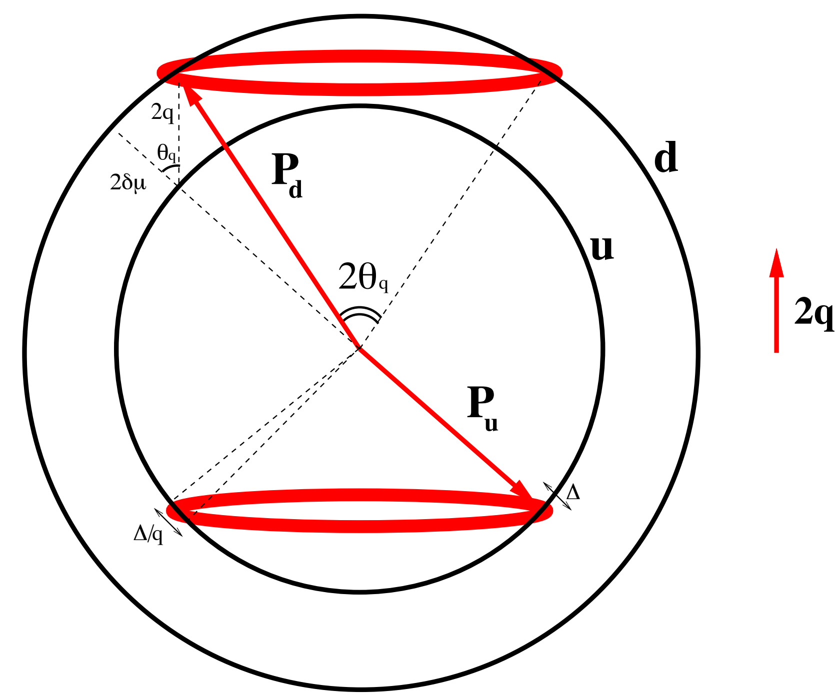

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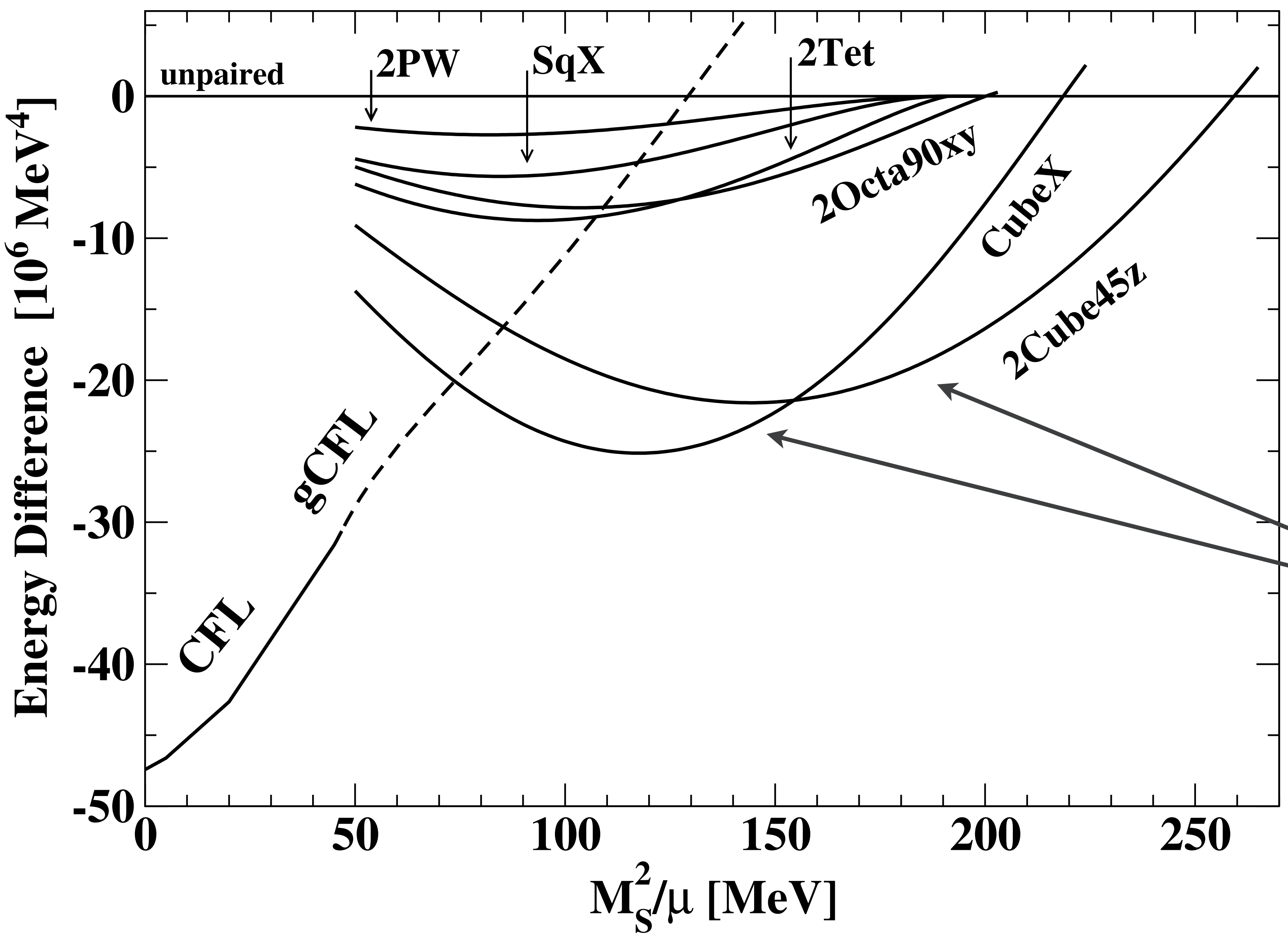
Deforming the Fermi sphere does not cost too much!
The free energy gain due to pairing overcompensates this co

Outlook

The IGL technique can be applied to the Crystalline Color Superconductors

NJL + GL expansion



Weird non-monotonic behavior

Rajagopal and Sharma Phys.Rev. D74 (2006) 094019

AN ASIDE:
GW ECHOES FROM STRANGE STARS

MM and F. Tonelli, (2018), arXiv:1805.02278 [gr-qc] to appear in Phys Rev D

GW echoes

Recent claim, J. Abedi and N. Afshordi, (2018), arXiv:1803.10454 [gr-qc], of a GW signal in the LIGO GW170817 post-merger data at a frequency

$$\boxed{f_{\rm echo} \approx 72 \text{ Hz}}$$

with a significance of $4.2\,\sigma$

Interpreted as GW echoes associated to a Planck-scale structure near the black hole horizon. If confirmed this may indicate quantum effects in GR.

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Alternative explanation:

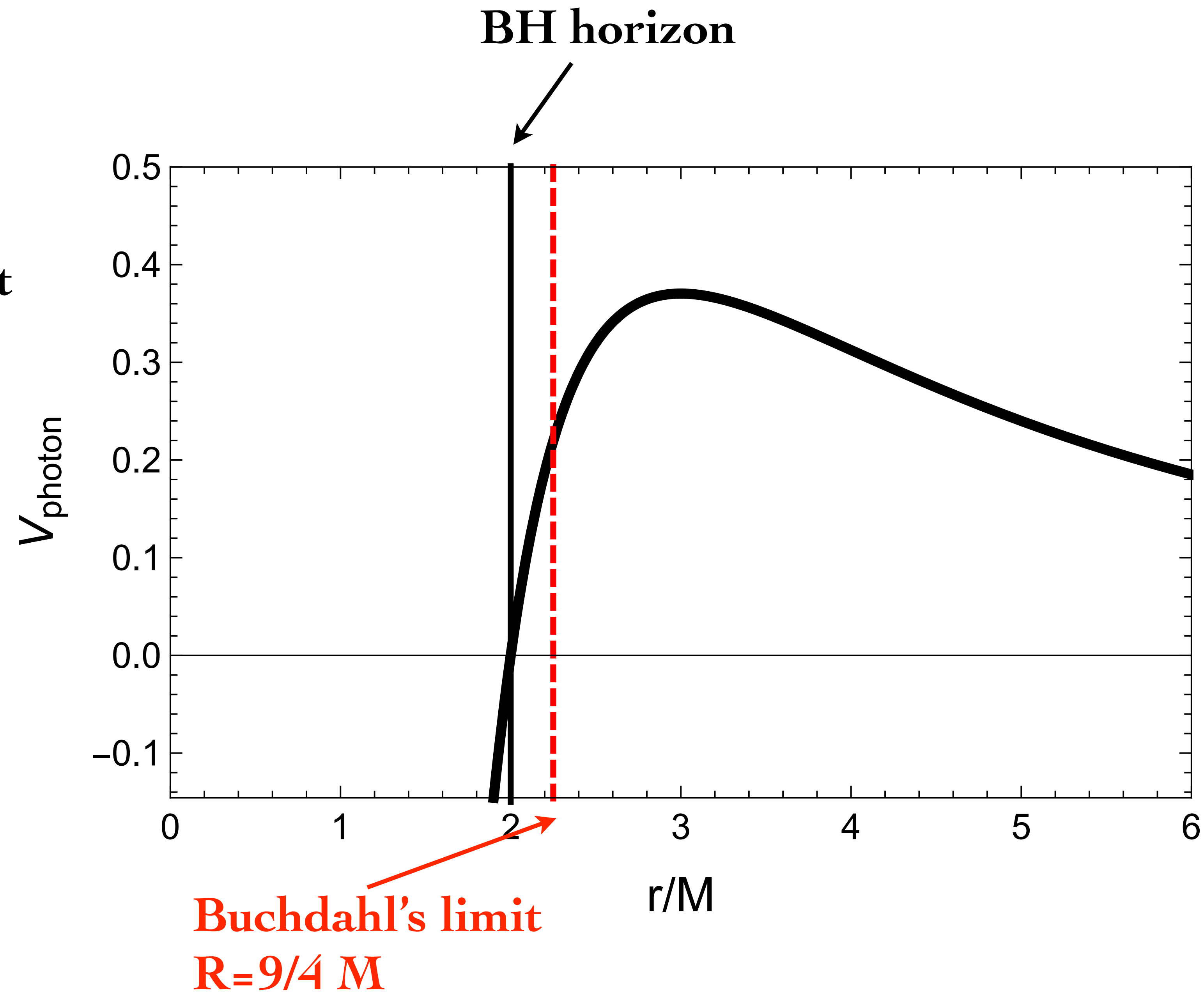
A signal of a ultracompact stellar object, very close to the Buchdahl's limit compactness
P. Pani and V. Ferrari, (2018), arXiv:1804.01444 [gr-qc].

In arXiv:1805.02278 [gr-qc] we tried to figure out whether a strange star can be ultracompact and emit GW echoes

GW echoes and ultracompact stars

Def.
Ultracompact stars are stars that have a photon-sphere, that is

$$9/4\,M < R < 3M$$



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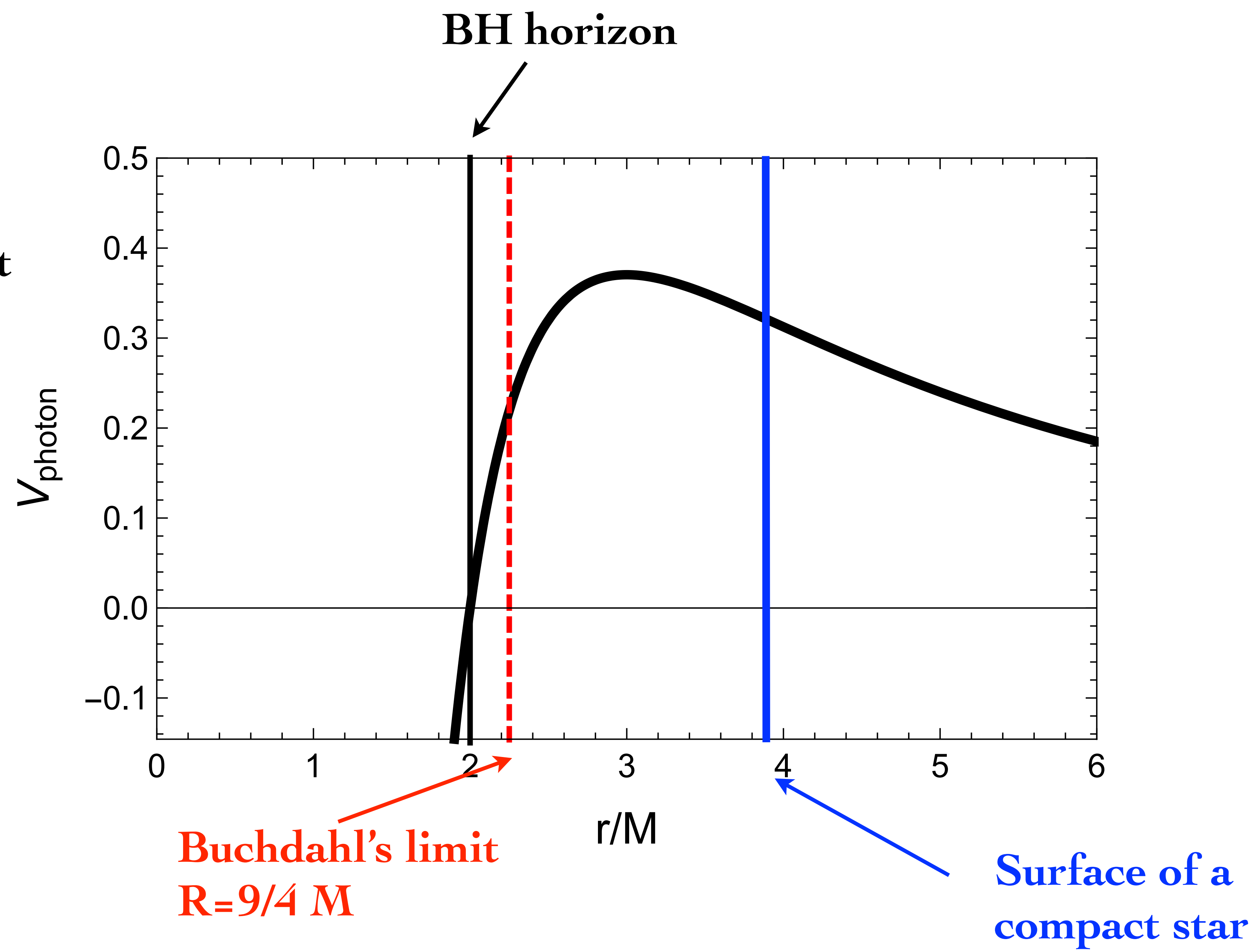

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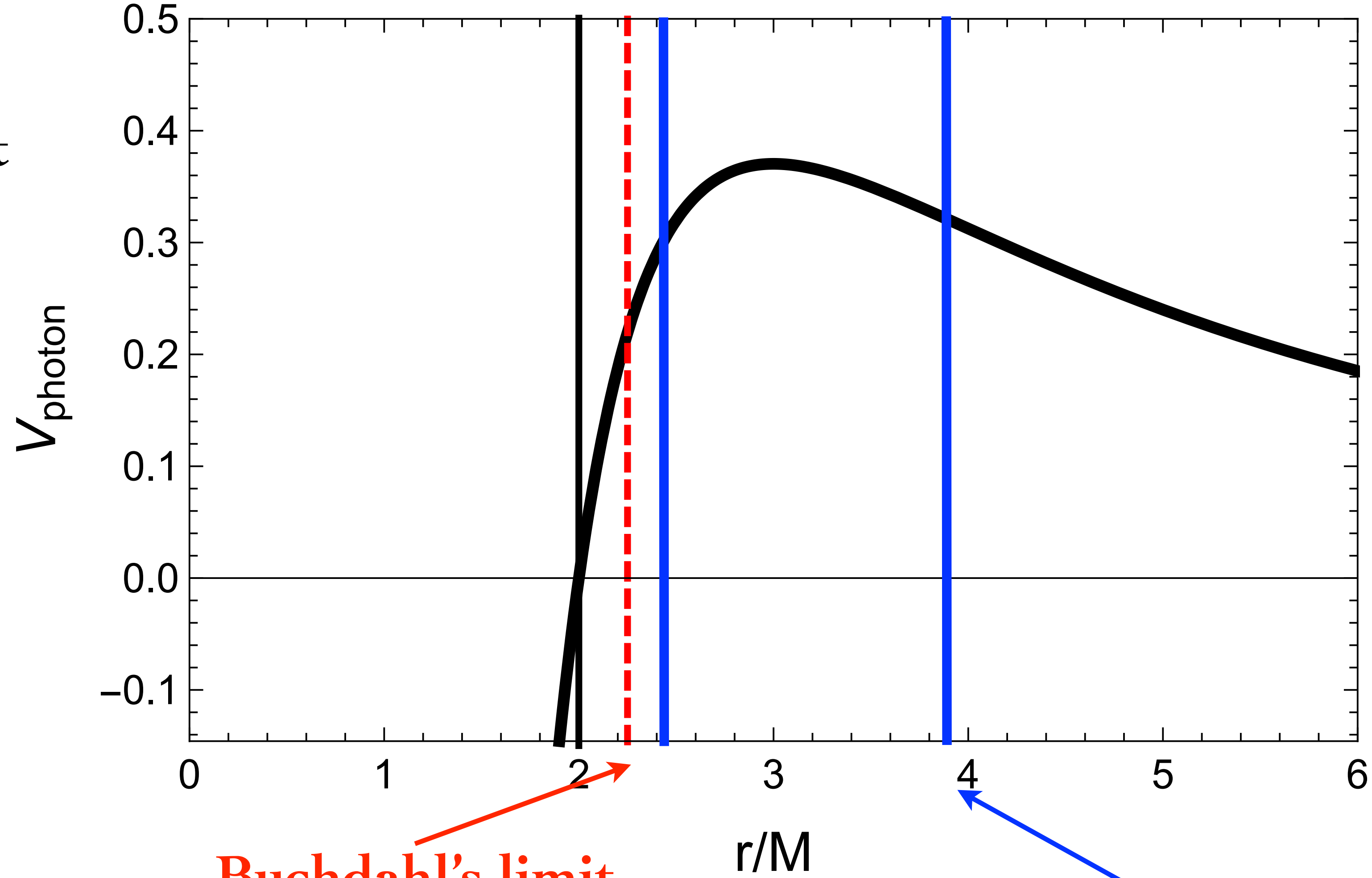
BH horizon

Surface of a ultracompact star



Buchdahl's limit
R=9/4 M

Surface of a compact star

GW echoes and ultracompact stars

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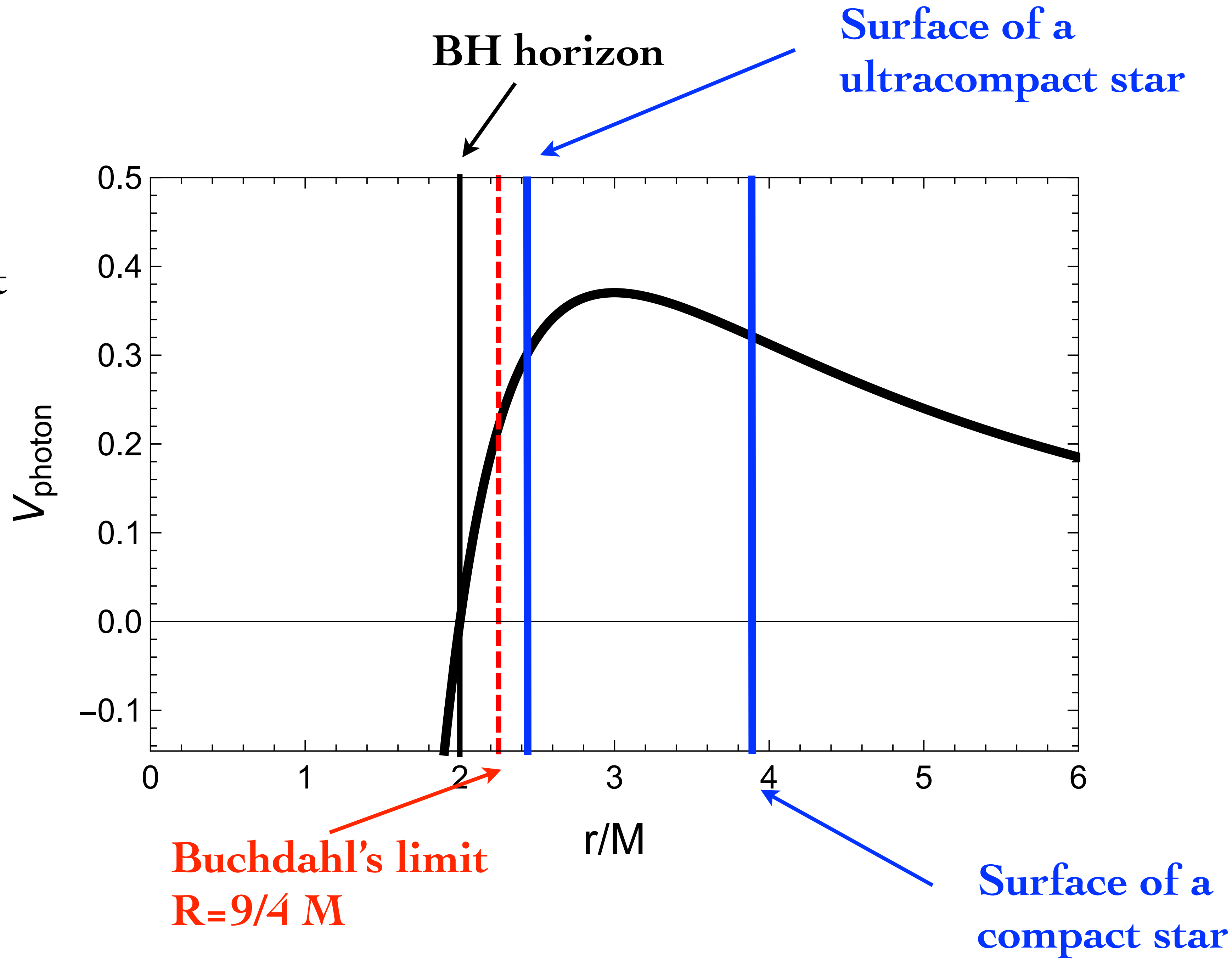
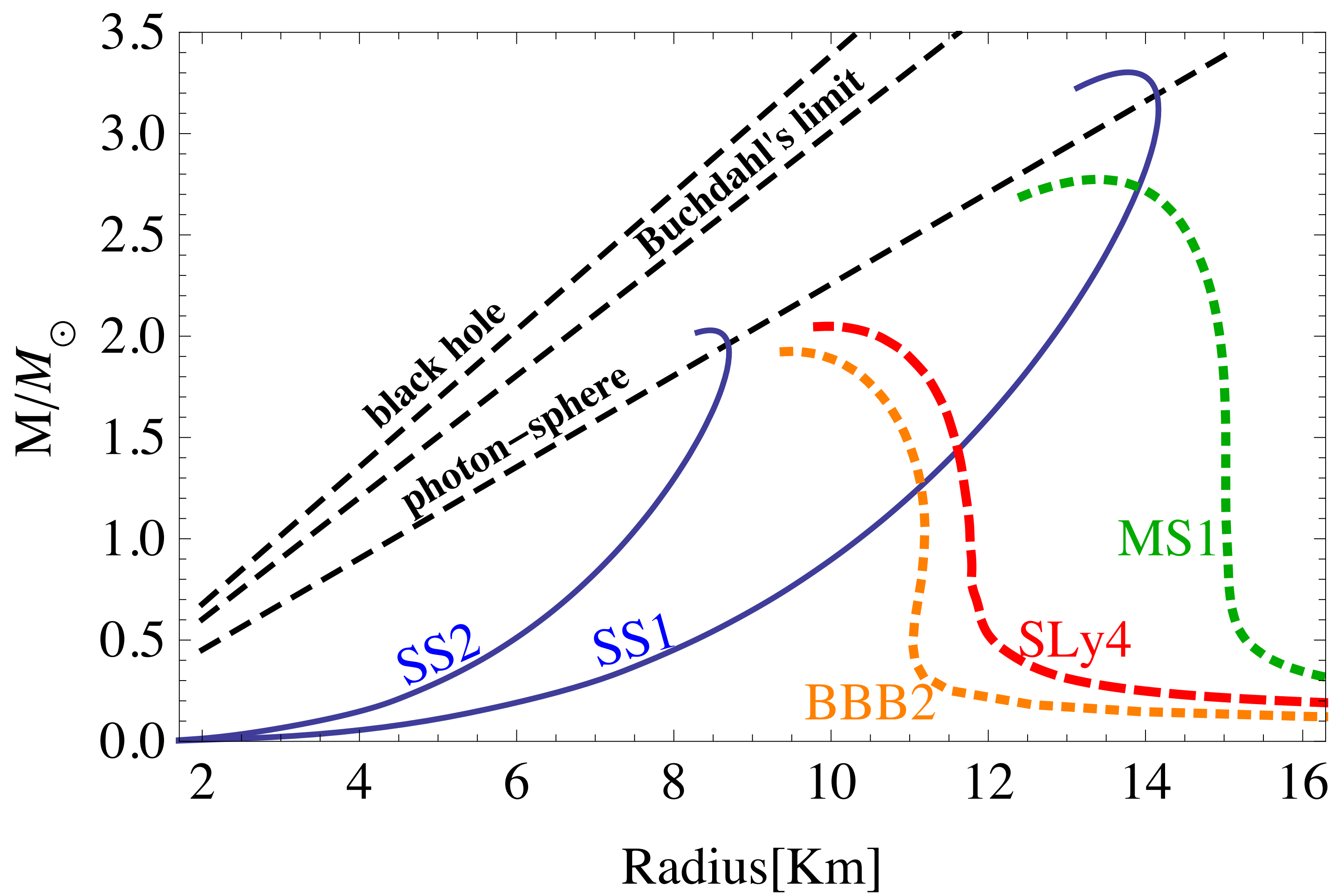


Typical echo time

$$\tau_{\rm echo} = \int_0^{3M} \frac{dr}{\sqrt{e^{2\Phi(r)}\left(1 - \frac{2m(r)}{r}\right)}}$$

Large for configuration close to R=2M

Strange ultracompact stars



$p = c_s^2(\epsilon - 4B)$ **we take** $c_s = 1$ **and** $B_1 = (145\ \mathrm{MeV})^4$, $B_2 = (185\ \mathrm{MeV})^4$

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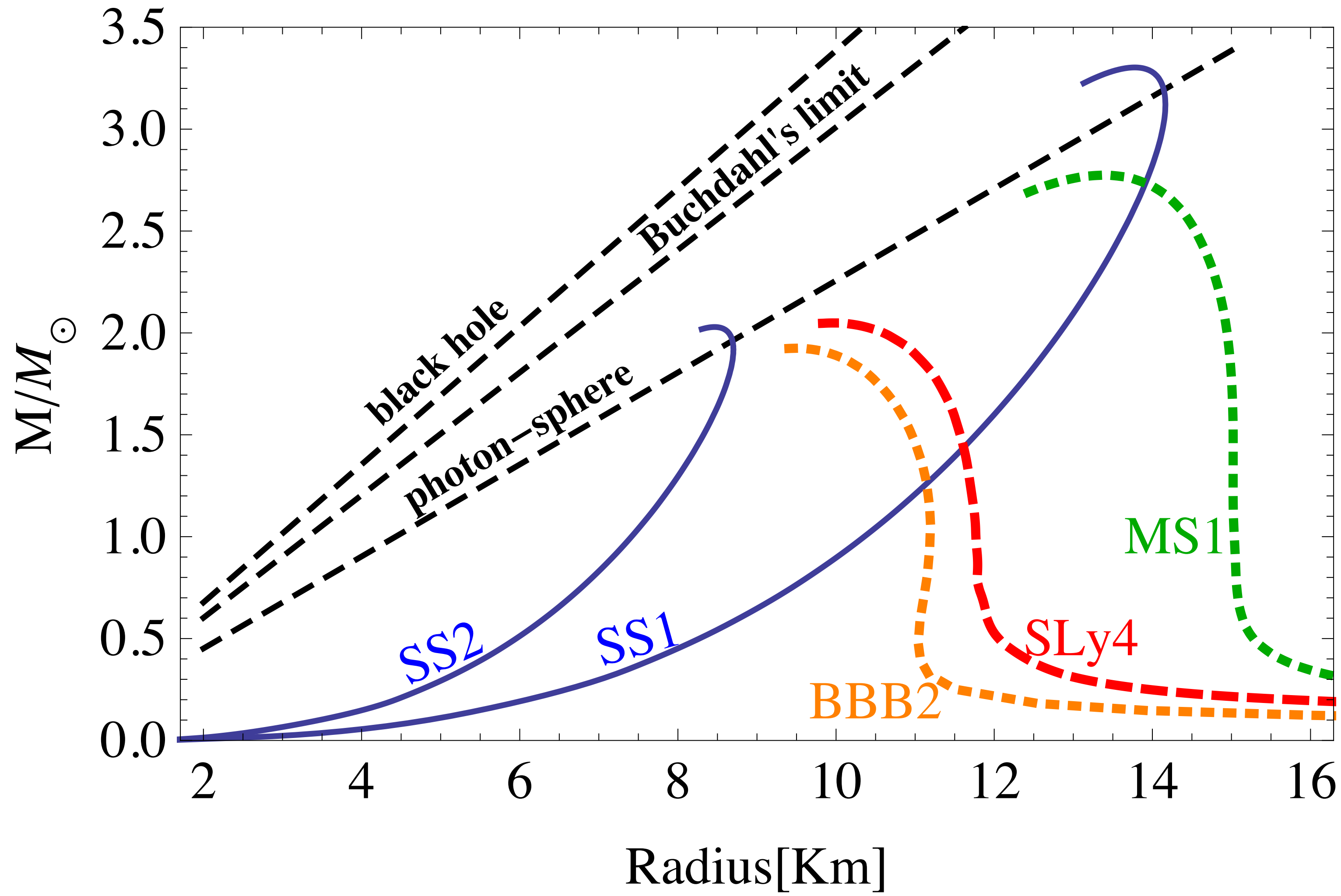

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Strange stars can have a photon-sphere but they hardly approach the Buchdahl's limit, thus

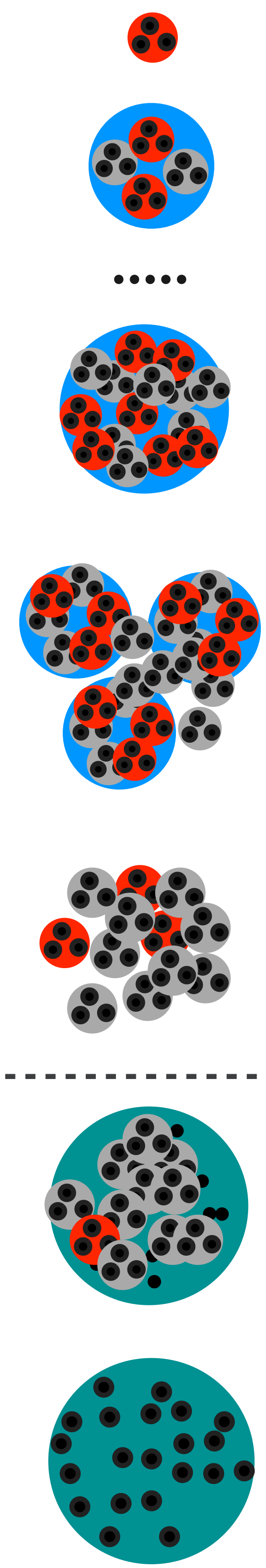
$$\omega = \pi/\tau_{\mathrm{echo}} = 10 - 17\ \mathrm{kHz}$$

Conclusions

- **We have proposed a novel GL expansion, which improves the description of the phase transitions to the inhomogeneous phases**
- **It requires the knowledge of one (semi-)analytical expression of the free energy**
- **We have applied it to the inhomogeneous chiral symmetry breaking**
- **Aside: GW echoes from strange stars are possible, but only at order 10 kHz frequency**

BACKUP

Increasing baryonic density



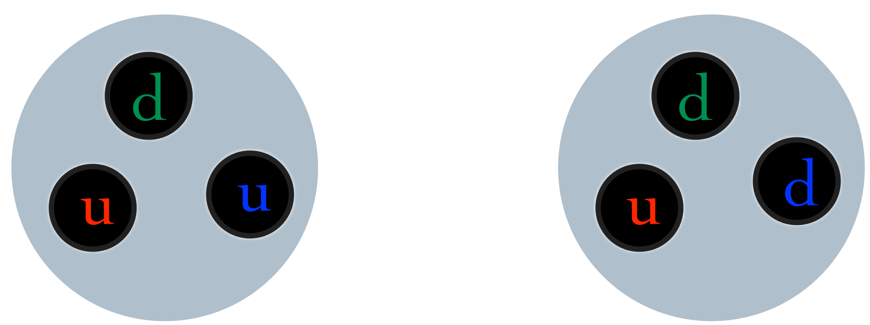
	Density	CSO part	$\alpha_s \equiv \alpha_s(\mu)$	Degrees of freedom
H				
He	......	atmosphere		light nuclei
......				.....
Fe	$10\ \mathrm{g\ cm^{-3}}$	outer crust	confining	heavy nuclei
neutron drip	$10^{11}\ \mathrm{g\ cm^{-3}}$	inner crust		
neutron proton soup	$10^{14}\ \mathrm{g\ cm^{-3}}$	core		neutrons and protons
quark drip	very large		strong coupling	quarks and gluons Cooper pairs of quarks?
quark soup	extreme		weak coupling	Cooper pairs of quarks NGBs

ρ

Quark model

BARYONS

proton neutron

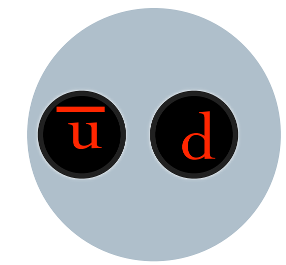


....

$$M_n \sim 1\text{GeV} \gg m_{u,d}$$

MESONS

pions



....

$$M_\pi \sim 135 \text{ MeV} \gg m_{u,d}$$

Quarks and gluons are the building blocks of hadrons

Q	quark flavor (mass in MeV)		
$+2/3$	u (3)	c (1300)	t (170000)
$-1/3$	d (5)	s (130)	b (4000)

The theory describing quarks and gluons is **Quantum Chromodynamics (QCD)**: a nonabelian SU(3) gauge theory.
Quarks form a triplet in the fundamental representation
Gluons are the vector gauge bosons associated to the octet adjoint representation