Equation of State (Quark Matter)

Equation of State (UDS)

We use the Bag model with a non-zero temperature for the equation of state (see section 4.2 in Niebergal's thesis),

$$\begin{aligned} P &= \frac{19}{36}\pi^2 T^4 + \frac{T^2}{2}\sum_i \mu_i^2 + \frac{1}{4\pi^2}\sum_i \mu_i^4 - B \\ h &= 4\,(P+B) \\ s &= \left(\frac{\partial P}{\partial T}\right)_{V,\mu} \\ n_i &= \frac{\mu_i^3}{\pi^{\ 2}} + \mu_i T^2 \ . \end{aligned}$$

Here, the index i ranges over all the quark species. In the expression for pressure one could readily add the electron pressure, however, it turns out to be negligible.

At the given densities the electrons are relativistic degenerate and so it is assumed that there are no charge imbalances. Hence, the electron density, needed to calculate neutrino production and slow reaction rates, is found by charge neutrality $n_e = n_u - n_{\mathrm{B}}$, where $n_{\mathrm{B}} = (n_u + n_d + n_s)\,/3$ is the total baryon density.

Equation of State (stiff UDS)

However, it turns out that the simplest version of the MIT Bag Model equations cannot sustain compact stars above two solar masses, which contradicts current observations (Demorest et al., 2010). On the other hand, it turns out that we can add a strong coupling constant ($0 < a_s < 1$; default 0.4) as, which stiffens the EoS. A stiffer EoS implies there is more pressure for a given density. Doing so we can get 2 solar mass stars:

$$P_f(a_s) = P_f - \left[\frac{7}{60}T^4\pi^2\frac{50a_s}{21\pi} + \frac{2a_s}{\pi}\left(\frac{1}{2}T^4\mu^2 + \frac{\mu^4}{4\pi^2}\right)\right],$$

where P_f is the EoS of quark matter for flavour f without perturbative corrections.

Equation of State (CFL)

In this situation matter (cells) below Tc cause matter in the region to release energy - freed as a result of quark pairing. The resulting discontinuity will form a superconducting interface, where on the superconducting side an extra term ($\propto \Delta$ the pairing energy gap) would be included in the equation-of-state.

Equation of State (Hadronic)

Equation of State (ud; special case)

The same equation-of-state is used for both the upstream (unburnt) and downstream (burnt) fluids. Even though the upstream fluid is nuclear matter, or some form of mixed matter, because the timescale for dissociation from a nucleus to u,d quarks is governed by the strong force it is negligible compared to the weak reaction timescales. Thus, at the point of burning it is reasonable to treat the nuclei as having already dissolved. Although the order of this phase transition is currently unknown, if it is a first-order transition an additional jump in pressure is being neglected. This extra jump in pressure would pre-shock the fluid upstream of the interface to some degree, changing some of the burning dynamics. A more accurate equation-of-state would be needed to determine the extent of this effect.

Equation of State (Hadronic; general)

Here, the user can enter either an analytical or a tabulated EoS which will be interpolated and/or extrapolated accordingly by Burn-UD.

EOSs implemented or to be implemented:

1. UD (default)
2. Hempel & Schaffner-Bielich (2010)
3. A. Akmal, V. R. Pandharipande and D. G. Ravenhall (1998)
4. J. M. Lattimer and F. D. Swesty (1991)
5. Database1: http://asphwww.ph.noda.tus.ac.jp/eos-gate/ (EOS DataBase; requires an account)
6. Database2: https://compose.obspm.fr/home. (Go to “Bibliography” and search; eg: “Akmal", “Lattimer” or "Schaffner-Bielich" to download data and papers)